CX 4240 Background Test

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1 Probability and Statistics 1

1.1 What are the median and mean of a set $\{4, 7, 10, 13, 6\}$, respectively?

A. $7, 8$ B. $8, 8$ C. $7, 9$ D. $8, 9$

1.2 Assume X is a random variable with the variance of $Var[X]$. What would be the variance of $aX + b + c$ where a, b, c are scalar constants?

1.3 Let X_1, $X_2, \ldots, X_n$ be a set of samples drawn independently and identically from a distribution $P(X)$ over a one dimensional random variable $X \in \mathbb{R}$. What is the formula to estimate the variance?

1.4 Suppose that given random variable X, random variables Y and Z are conditionally independent. Which one of the following is true?

A. $P(Y|X, Z) = P(Y|Z)$ B. $P(Y|X) = P(Z|X)$ C. $P(Y|Z, X) = P(Y|X)$

1.5 Assume that the cumulative distribution function (C.D.F.) of a random variable is given as follows. Write down the expression for the probability density function (p.d.f.) of X.

$$F(x) = \begin{cases} 0 & if \quad x < 0 \\ x & if \quad 0 \leq x < 1 \\ 1 & if \quad x \geq 1 \end{cases}$$

1.6 One box has several different colored balls inside of it. It has 1 red, 2 green, 4 blue, and 8 yellow balls. All the balls are the same size and shape. If you take out a ball from the box without looking, what is the probability that it will NOT be a yellow ball.

A. $\frac{8}{15}$ B. $\frac{7}{8}$ C. $\frac{7}{15}$ D. $\frac{6}{15}$

1.7 Which of the following statements is true? (Notation $A \cup B$ means "either event A or B happens"; notation $A \cap B$ means "both event A and B happened simultaneously").

A. If the events A and B are independent, then $P(A \cup B) = P(A) + P(B)$
B. $P(A \cup B) < P(A) + P(B)$
C. If A and B are mutually exclusive, then $P(A \cap B) = P(A)P(B)$
D. None of the above.

1.8 Suppose we throw two dice (each with 6 sides) simultaneously. Assuming the two dices are independent and each side comes up with equal probability. What is the probability of getting sum as 8 of the two numbers shown?

1.9 Suppose that $f(x)$ is the probability density function (p.d.f.) of random variable X. Then, is $f(x|b < x \leq a) = \frac{f(x)}{F(a)-F(b)}$ for any $b < x \leq a$ True or False?

1.10 Assume that we have m coins. We toss each one of them n times. The probability of heads showing up for each coin is p. What's the probability of getting n heads for some coin? Your answer should be in terms of m, n and p.

2 Probability and Statistics 2

2.1 Suppose that there are two events A and B, and furthermore one event is included in another event: $A \subseteq B$ such that $P(A) > 0$ and $P(B) > 0$. How many of the following statements are true: **(1)** $P(B|A) = 1$, **(2)** $P(A|B) = P(A)/P(B)$?

A. 0 B. 1 C. 2

2.2 Let $Y_1, Y_2, \ldots, Y_n$ be a set of random sample of size n drawn independently and identically from a normal distribution with mean μ and variance σ^2. Then let $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. How many of the following statements are true: **(1)** $\overline{Y}$ is normally distributed, **(2)** mean $\mu_{\overline{Y}} = \mu/n$, and **(3)** variance $\sigma^2_{\overline{Y}} = \sigma^2/n$.

A. 0 B. 1 C. 2 D. 3

2.3 Is the following statement TRUE or FALSE: If we perform the same experiment a large number of times and the results are independent and identically distributed, then the average of the results obtained from a large number of trials should be close to the expected value, and will tend to become closer as more trials are performed.

2.4 Suppose random variables Y, x and ϵ are related by $Y = \beta_0 + \beta_1 x + \epsilon$, with β_0 and β_1 are parameters and ϵ is assumed to independent of x and follow normal distribution with mean 0 and constant variance. Please calculate: **(1)** $E(\epsilon|x)$, and **(2)** $E(Y|x)$.

2.5 Is the following statement TRUE or FALSE: $p\left(X = x|Y = y\right) = \frac{p(X=x)p(Y=y|X=x)}{\sum'_x p(X=x')p(Y=y|X=x')}$

2.6 The population correlation coefficient $\rho_{X,Y}$ between two random variables X and Y with expected values μ_X and μ_Y and standard deviations σ_X and σ_Y is defined as: $\rho_{X,Y} = \mathrm{corr}(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$. How many of the following statements are true: **(1)** X, Y independent $\Rightarrow \rho_{X,Y} = 0$ **(2)** $\mathrm{corr}(X,Y) \neq \mathrm{corr}(Y,X)$

A. 0 B. 1 C. 2

2.7 Is the following statement TRUE or FALSE: If X and Y are ***any*** two random variables and a and b are ***any*** two constants, then $Var(aX + bY) = a^2 Var(X) + b^2 Var(Y)$?

2.8 Suppose Y_1 and Y_2 are any two random variables, the ***conditional expectation*** of $g(Y_1)$, given that $Y_2 = y_2$, is defined to be: **(1)** $E\left(g(Y_1)|Y_2 = y_2\right) = \int_{-\infty}^{+\infty} g(y_1) f(y_1|y_2)\, dy_1$, or **(2)** $E\left(g(Y_1)|Y_2 = y_2\right) = \int_{-\infty}^{+\infty} g(y_1) f(y_1|y_2)\, dy_2$

A. (1) B. (2)

2.9 Suppose $F(y)$ is a cumulative distribution function over real domain, then how many of the following statements are TRUE: **(1)** $F(-\infty) = 0$, **(2)** $F(+\infty) = 0$, and **(3)** If y_1 and y_2 are any values such that $y_1 < y_2$, then $F(y_1) \leq F(y_2)$?

A. 0 B. 1 C. 2 D. 3

2.10 **Suppose $X \sim Normal\,(5, 3^2)$. What is $P\,(X \leq 8)$ in terms of the standard normal variable Z? (Or $P(X \leq 8)$ is equal to?)**

A. $P\,(Z \leq 1)$ B. $P\,(Z \leq -1)$ C. $P\,(Z \leq 0.6)$

3 Linear Algebra

3.1 **We have two matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times d}$. Let A_{ij} be the entry of A in the i-th row and j-th column, and similar notation applies for B_{ij}. Let the multiplication of A and B be $C = AB$, what is the dimensions of matrix C and how do you compute entry C_{ij}?**

3.2 **If we have $a + 2b - 3c = 0$, where $a, b, c \in \mathbb{R}^n$ are vectors, then these vectors are linearly independent. True or False?**

3.3 **The solution for the linear system $Ax = b$, where $A \in \mathbb{R}^{m \times n}$ and $m \neq n$, $x \in \mathbb{R}^n$, and $b \in \mathbb{R}^m$ is $x = A^{-1}b$. True or False?**

3.4 **If x is in the null space of A, then**

A. $Ax = 0$ B. $Ax = x$ C. $x^T A = 0$ D. $x^T A = x^T$

3.5 **If Q is an orthonormal matrix, which of the following statements are correct? Choose all that apply**

A. $QQ = I$ B. $QQ^T = I$ C. $Q^T Q = I$ D. $Q = Q^{-1}$ E. $Q^T = Q^{-1}$

3.6 **The squared Frobenius norm $\|X\|_F^2$ can also be expressed as**

A. $\mathrm{tr}(X^T X)$ B. $\det(X^2)$ C. $XX^{-1}X$

3.7 **What is the rank and trace of the matrix A**

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & -1 \\ 0 & 6 & -2 \end{pmatrix}$$

3.8 **Which vectors are in the column space of the matrix A in Question 3.7? Choose all that apply.**

A. $[1, 2, 0]^T$ B. $[1, 0, -2]^T$ C. $[0, 0, 0]^T$ D. $[1, 3, 4]^T$ E. $[0, 2, 0]^T$

3.9 **Suppose $A \in \mathbb{R}^{n \times n}$ is a symmetric matrix, and it has n eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n \in \mathbb{R}$ and the corresponding n orthonormal eigenvectors $v_1, v_2, \ldots, v_n \in \mathbb{R}$, which one of the following is true? (choose one)**

A. $Av_1 = \lambda_1 v_1$ B. $A = \sum_{i=1}^n \lambda_i v_i v_i^\top$ C. Both A and B. D. None of the above.

3.10 **Let $f(x) = a^\top x$, and $g(x) = (a^\top x)^2$ where $x \in \mathbb{R}^n$ and $a \in \mathbb{R}^n$. Write the derivatives of $f(x)$ and $g(x)$ with respect to x in matrix and vector format, i.e., $\nabla f(x)$ and $\nabla g(x)$.**

4 Matlab/Numpy: Answer with Matlab or numpy commands.

If you are answering in python, assume that the following import statement is available (from numpy import *)
The questions are set in matlab but the following conversions can be used for python. For the matlab version of the code, given in the left column, the right column denotes the equivalent python version

```
[1 2 3] <--> array([1,2,3])
[1 2; 3 4] <--> array([[1,2],[3,4]])
A' <--> A.T
A*b <--> A*b (for scalar b)
A.*B <--> A*B
```

4.1 **Create a vector of all the even numbers between 31 and 75.**

4.2 **Let x = [2 5 1 6]. Compute the square root of each element, and assign the result to a.**

4.3 **Let x = [3 2 6 8]$'$ and y = [4 1 3 5]$'$. Answer in Matlab commands. Raise each element of x to the power specified by the corresponding element in y.**

4.4 **Given the array A = [2 4 1 ; 6 7 2 ; 3 5 9], assign the first row of A to a vector called a.**

4.5 **Given x = [3 15 9 12 -1 0 -12 9 6 1], set the values of x that are positive to zero.**

4.6 **Given the following code, calculate the result of w:**

```
if z < 5
        w = 2*z
elseif z < 10
        w = 9 - z
elseif z < 100
        w = sqrt(z)
else
        w = z
end
```

1. $z = 1$, w = ?
   A. 2
   B. 8
   C. sqrt(1)
   D. 200
2. $z = 9$, w = ?
   A. 18
   B. 0
   C. sqrt(9)
   D. 200
3. $z = 60$, w = ?
   A. 120
   B. -51
   C. sqrt(60)
   D. 200
4. $z = 200$, w = ?
   A. 400
   B. -191
   C. sqrt(200)
   D. 200

4.7 **Given A = [1 2 3; 4 5 6], b = 7 and C=A'*b, what is the result of C?**

4.8 **Given A = [1 2 3; 4 5 6], B = [9 8 7; 6 5 4] and C=A.*B, what is the result of C?**

4.9 **Given A = [3 6 2 5 9] and B = [9 5 3 7 1], what is the vector returned by command $A <= B$?**

4.10 **Given the vector x = [1 8 3 9 0 1], add up the values of the elements of x, and assign the result to a.**