Georgia Tech
College of Computing

CX4240 Spring 2026
Recurrent Neural Networks

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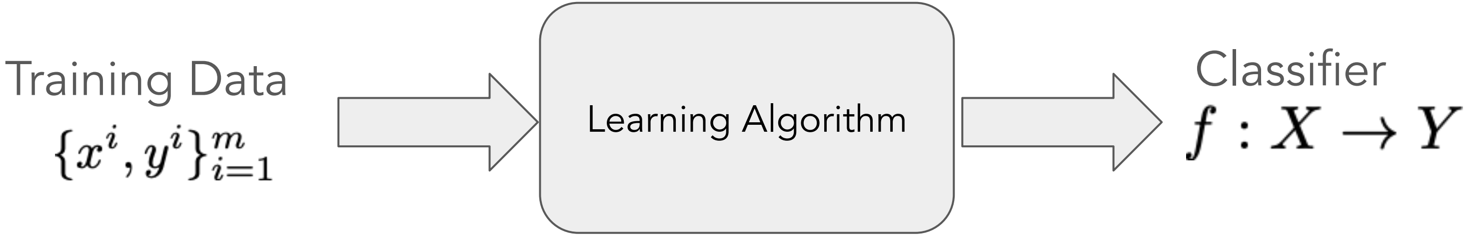
Based on Fei-Fei Li & Ehsan Adeli's Slides

Multiclass Logistic Regression Algorithms

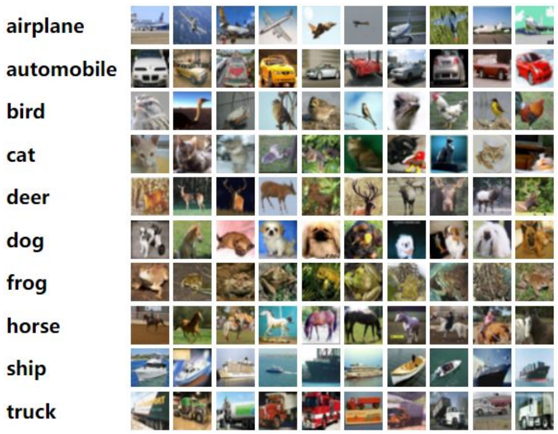


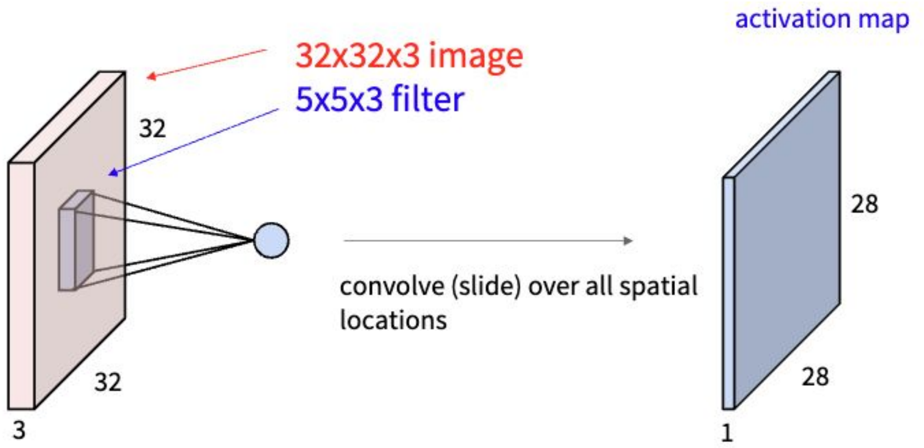
Multiclass Classification $Y \in \{0, 1, \ldots, k\}$

Multiclass Logistic Regression Pipeline

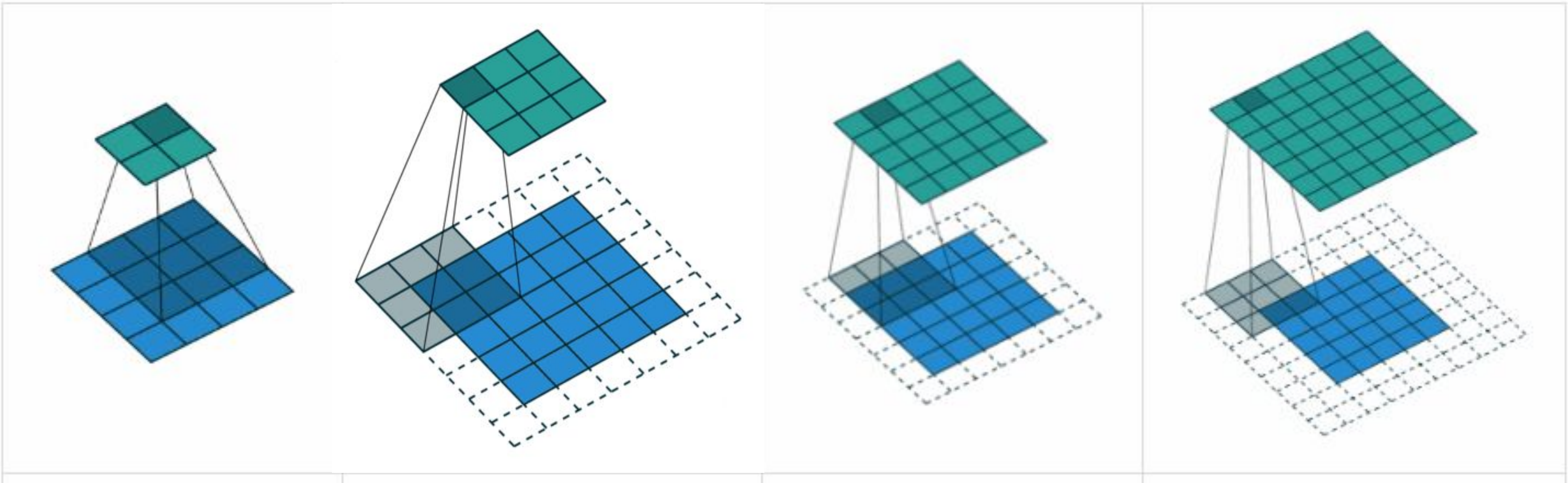
1. Build probabilistic models:
   Categorical Distribution + Conv NN
2. Derive loss function: MLE and MAP
3. Select optimizer: (Stochastic) Gradient Descent

Revisit Convolution Neural Networks





Convolution for 2D Images

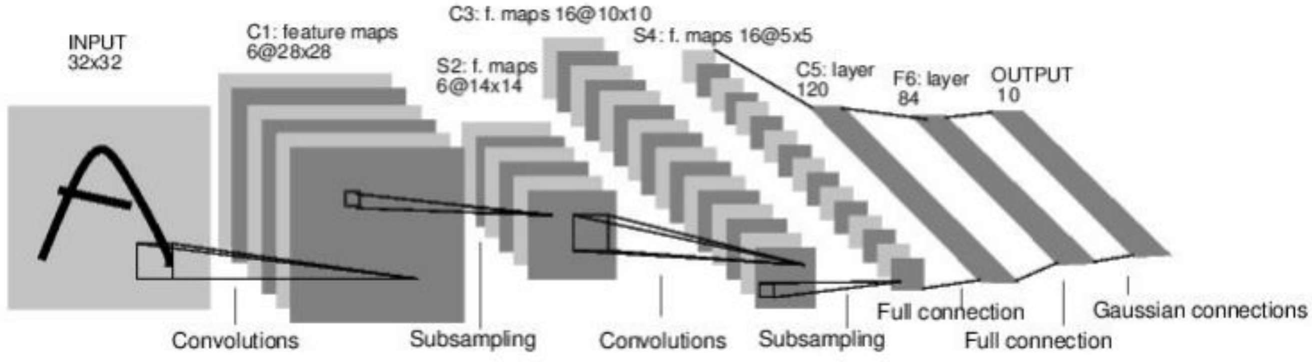


padding = 0, stride = 1 | padding = 1, stride = 2 | padding = 1, stride = 1 | padding = 2, stride = 1

$$W_{out} = \frac{W - F + 2P}{S} + 1$$

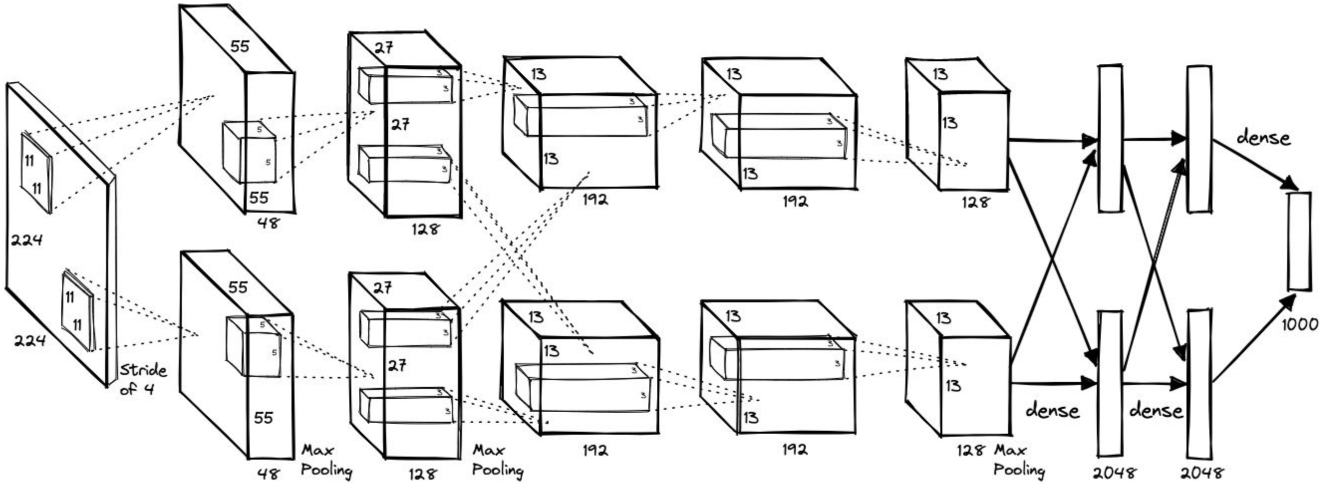
Animation from [Hochschule der Medien]

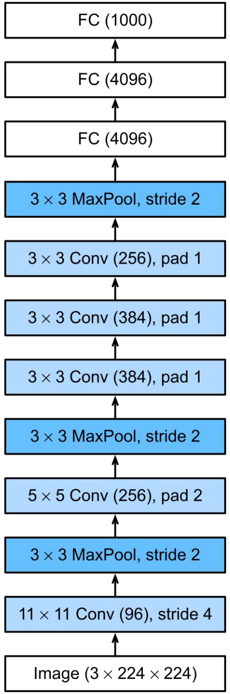
Put Everything Together for Images



[LeNet-5, LeCun 1980]

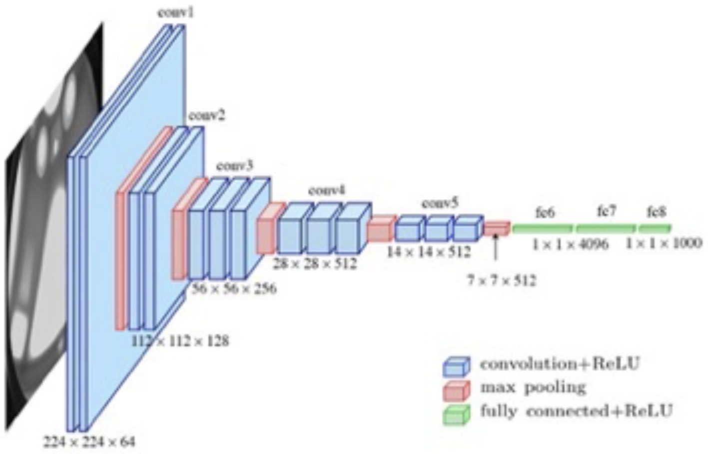
AlexNet (2012)

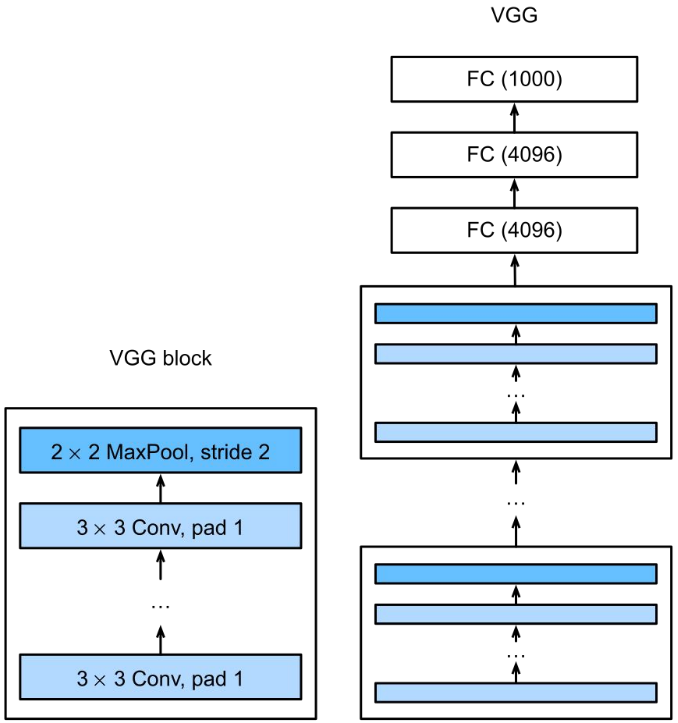




VGGNet (2014)

- Very Deep CNN
- With only 3*3 conv filters
  - Fewer parameters, deeper nonlinear layers





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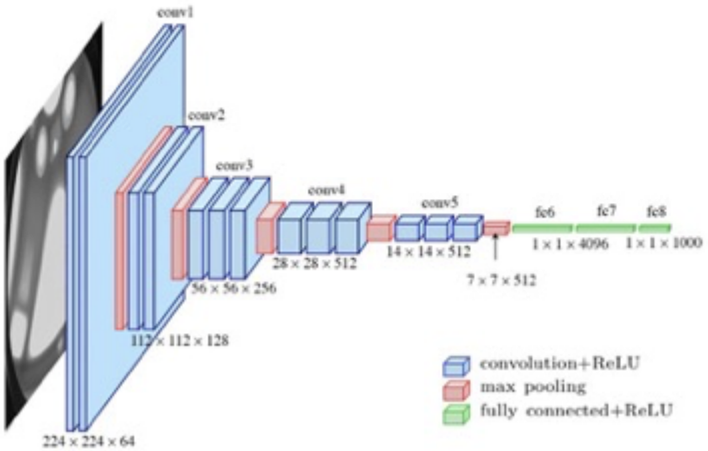

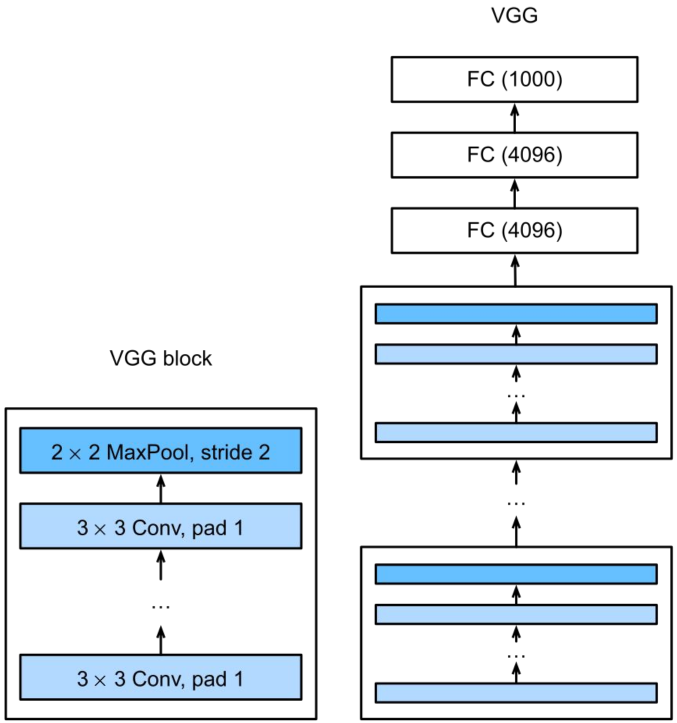

Keep increasing the depth?

VGGNet (2014)

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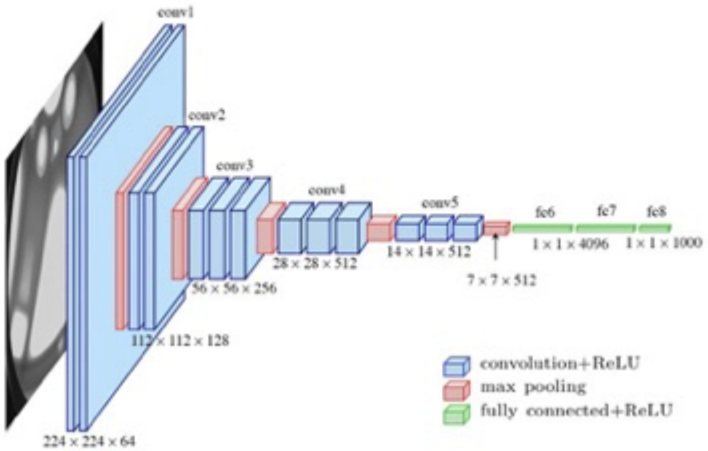

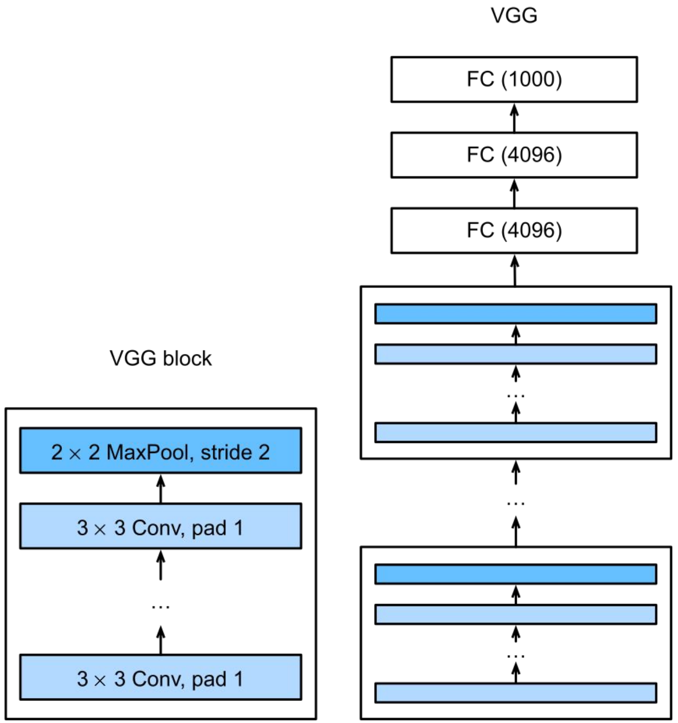

vanishing or exploding

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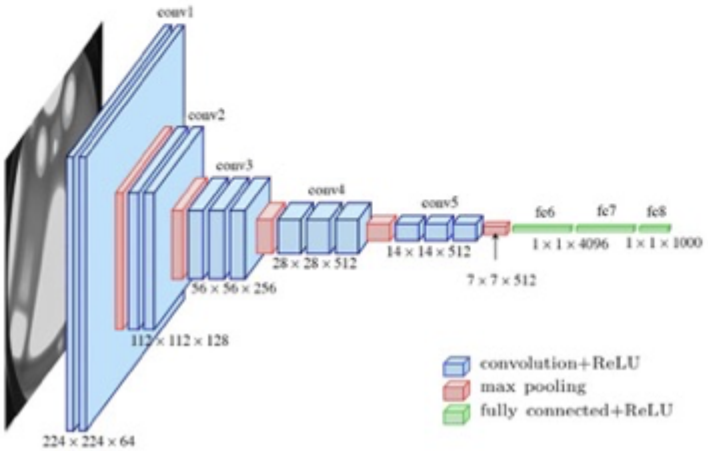

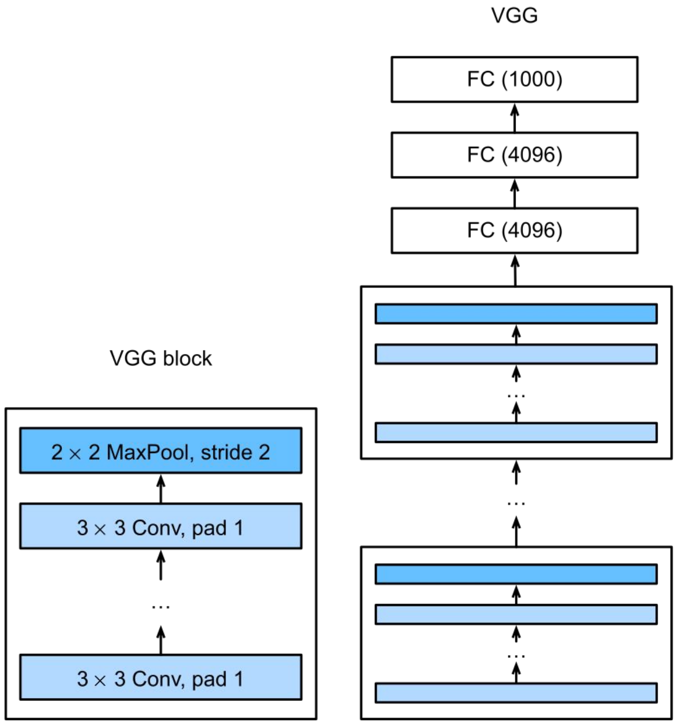

vanishing or exploding

$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial a_n} \cdot \frac{\partial a_n}{\partial a_{n-1}} \cdot \frac{\partial a_{n-1}}{\partial a_{n-2}} \cdot \cdots \cdot \frac{\partial a_{i+1}}{\partial a_i} \cdot \frac{\partial a_i}{\partial w_i}$$

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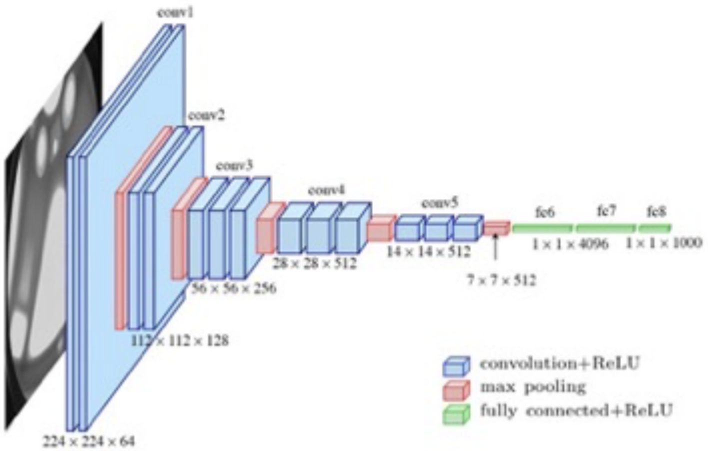

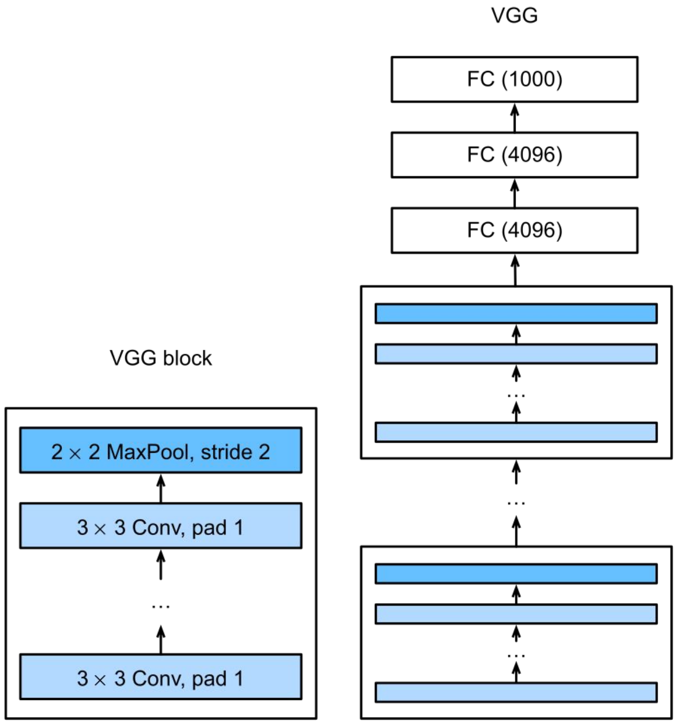

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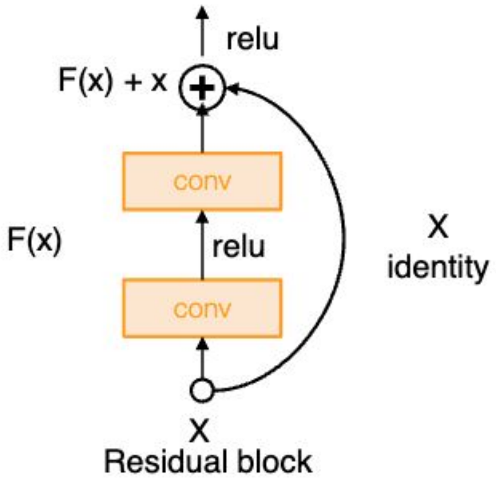
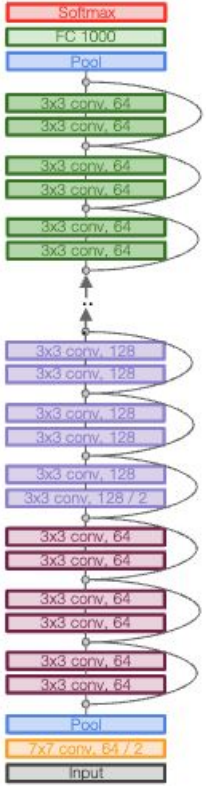
ResNet (2015)

- Very Deep CNN with residual connections
  - 152-layer model for ImageNet
  - ILSVRC’15 classification winner (3.57% top 5 error)
  - Swept all classification and detection competitions in ILSVRC’15 and COCO’15!





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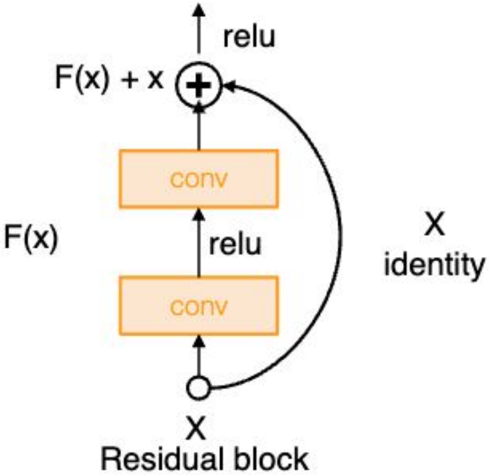
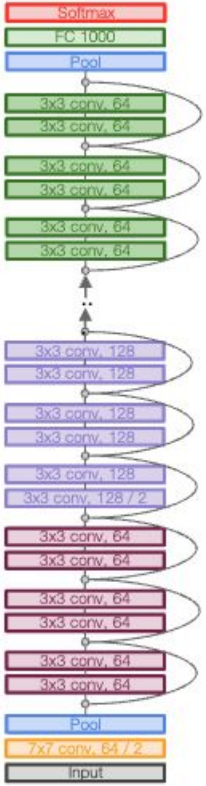
- Migrate gradient exploding and vanishing

$$y = x + F(x)$$

$$\frac{\delta E}{\delta x} = \frac{\delta E}{\delta y} * \frac{\delta y}{\delta x} = \frac{\delta E}{\delta y} * (1 + F'(x))$$
$$= \frac{\delta E}{\delta y} + \frac{\delta E}{\delta y} * F'(x)$$



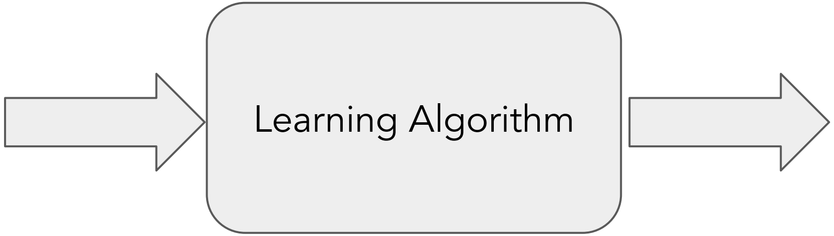
$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial a_n} \cdot \prod_{j=i}^{n} \frac{\partial a_j}{\partial a_{j-1}}$$



Multiclass Logistic Regression Algorithms

Training Data

$$\{x^i, y^i\}_{i=1}^{m}$$



Classifier

$$f : X \rightarrow Y$$

Multiclass Classification $Y \in \{0, 1, \ldots, k\}$

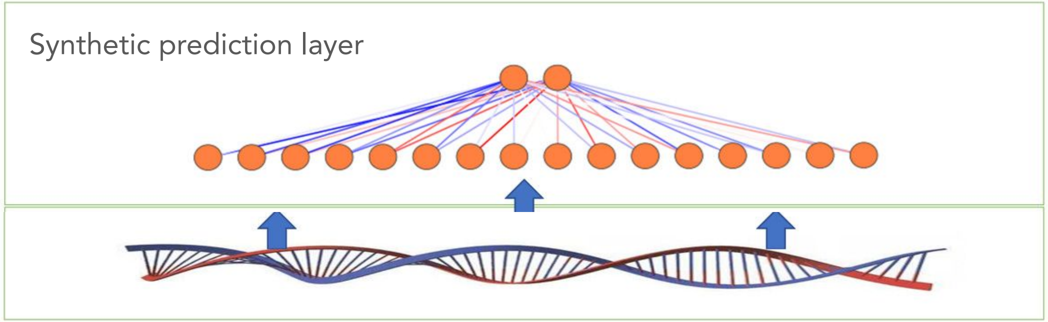
Multiclass Logistic Regression Pipeline

1. Build probabilistic models:
   Categorical Distribution + Conv NN
2. Derive loss function: MLE and MAP
3. Select optimizer: (Stochastic) Gradient Descent

Sequence Prediction

texts[0]
For a movie that gets no respect there sure are a lot of memorable quotes listed for this gem. Imagine a movie where Joe Piscopo is actually funny! Maureen Stapleton is a scene stealer. The Moroni character is an absolute scream. Watch for Alan "The Skipper" Hale jr. as a police Sgt.

Sequence Prediction



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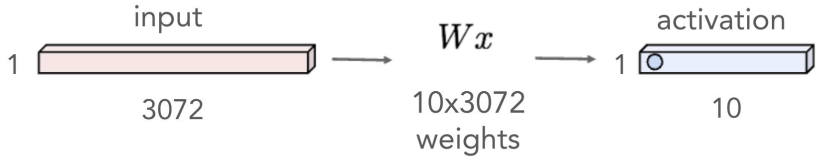
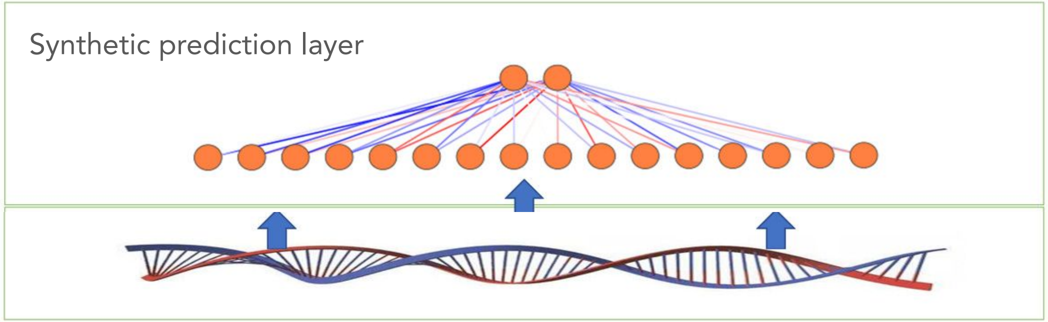


Sequence Prediction



I saw the movie with two grown children. Although it was not as clever as Shrek, I thought it was rather good. In a movie theatre surrounded by children who were on spring break, there was not a sound so I know the children all liked it. There parents also seemed engaged. The death and apparent death of characters brought about the appropriate gasps and comments. Hopefully people realize this movie was made for kids.

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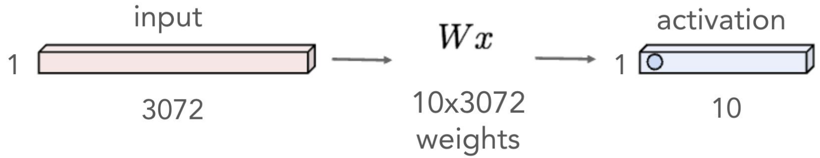

What if the length of sequences varies?

Sequential Regression Algorithms

Training Data

$\{\boxed{x^i}, y^i\}_{i=1}^{m}$

Learning Algorithm

Predictor

$f : X \to Y$

$Y \in \mathbb{R}$

Linear Regression Pipeline

1. Build probabilistic models:
   Gaussian Distribution + RNN
2. Derive loss function: MLE and MAP
3. Select optimizer: (Stochastic) Gradient Descent

Sequential Binary Classification Algorithms

Training Data

$\{\boxed{x^i}, y^i\}_{i=1}^m$

Learning Algorithm

Predictor

$f : X \to Y$

Binary Classification

$Y \in \{0, 1\}$

Binary Logistic Regression Pipeline

1. Build probabilistic models:
   Bernoulli Distribution + RNN
2. Derive loss function: MLE and MAP
3. Select optimizer: (Stochastic) Gradient Descent

Sequential Multiclass Logistic Regression Algorithms

Training Data

$\{\boxed{x^i}, y^i\}_{i=1}^{m}$

Learning Algorithm

Classifier

$f: X \rightarrow Y$

Multiclass Classification $Y \in \{0, 1, \ldots, k\}$

Multiclass Logistic Regression Pipeline

1. Build probabilistic models:
   Categorical Distribution + RNN
2. Derive loss function: MLE and MAP
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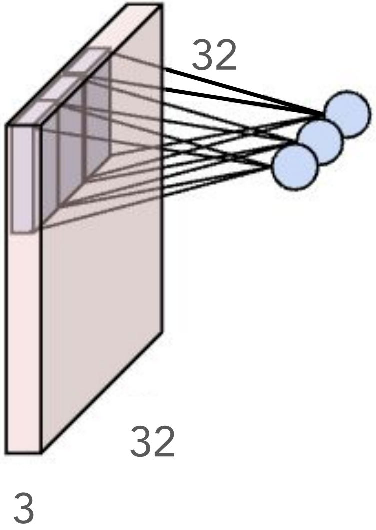
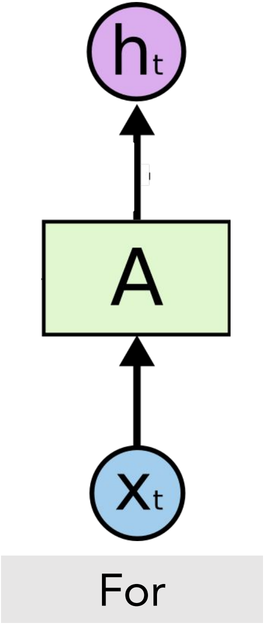
Inspiration from Convolution Layer



one computation cell is shared

Recurrent Neural Network



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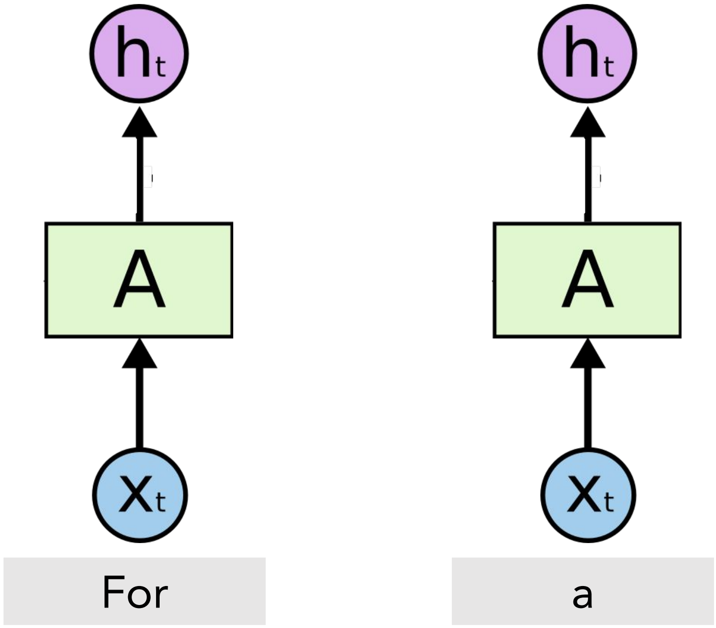

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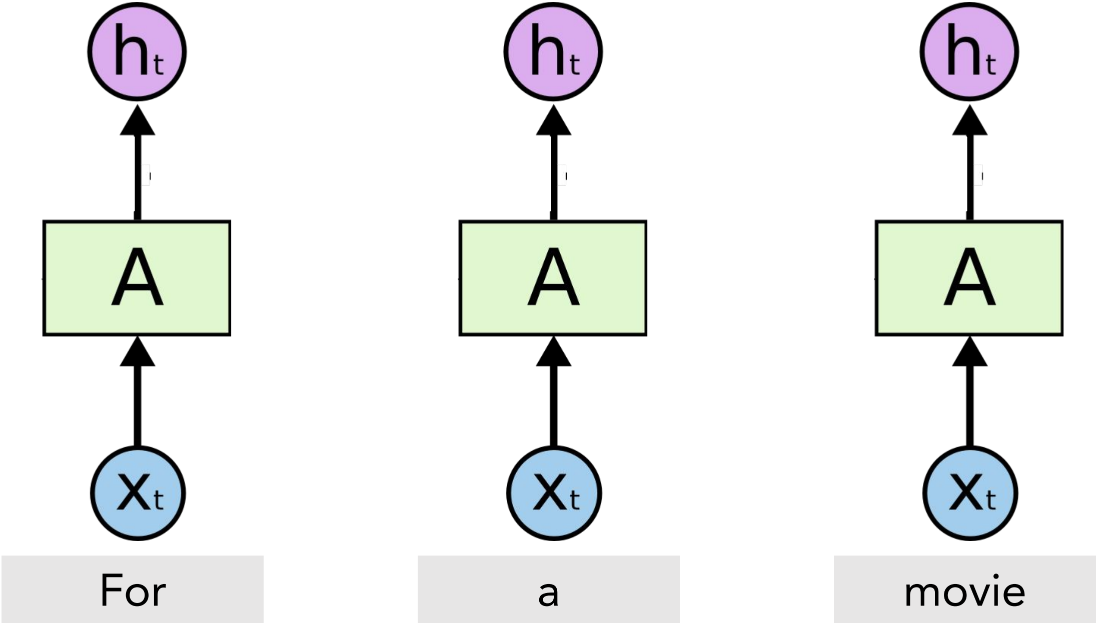

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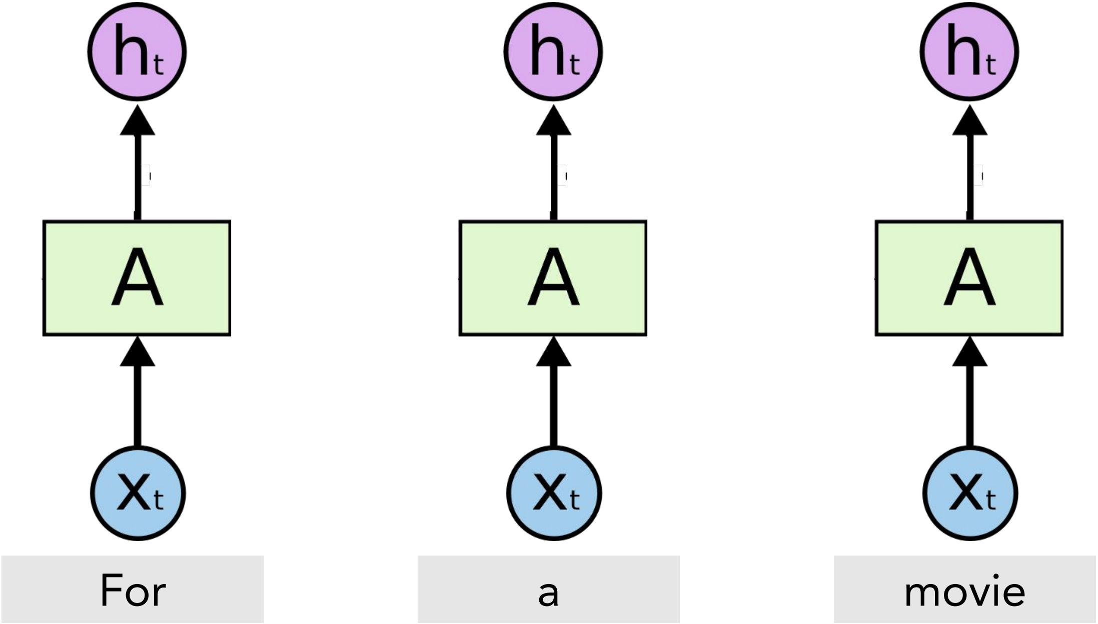

Order Matters!

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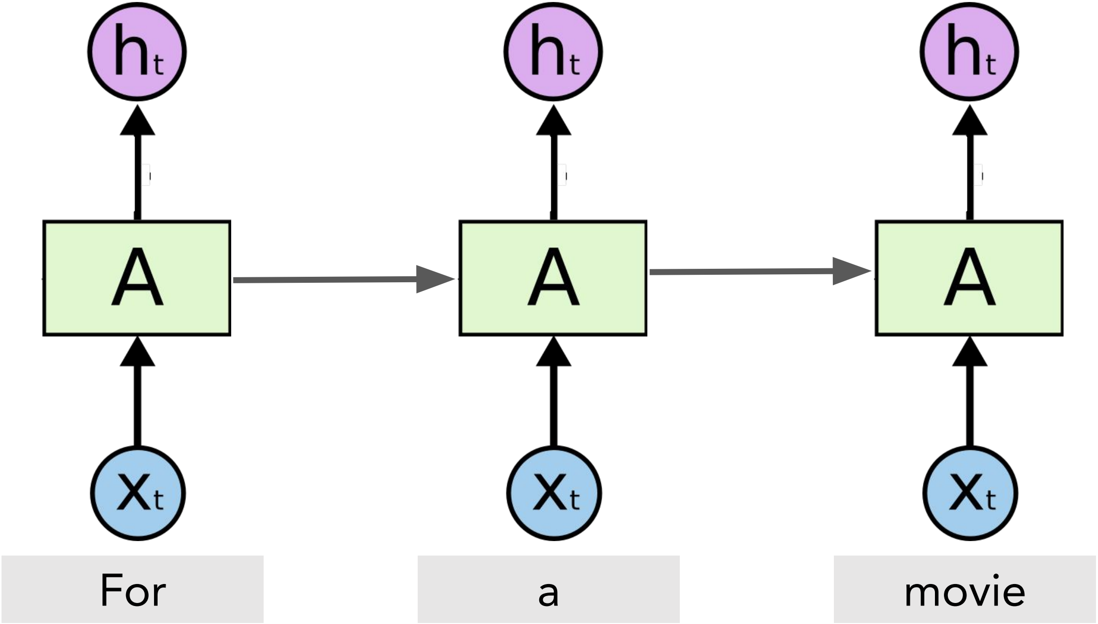

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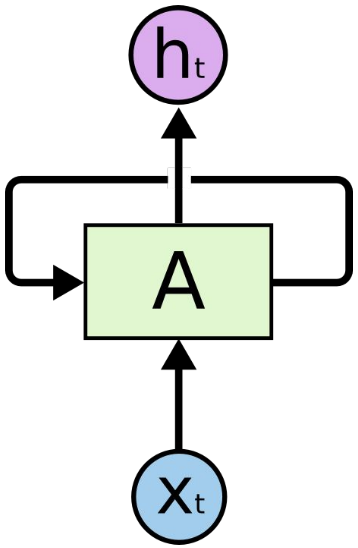

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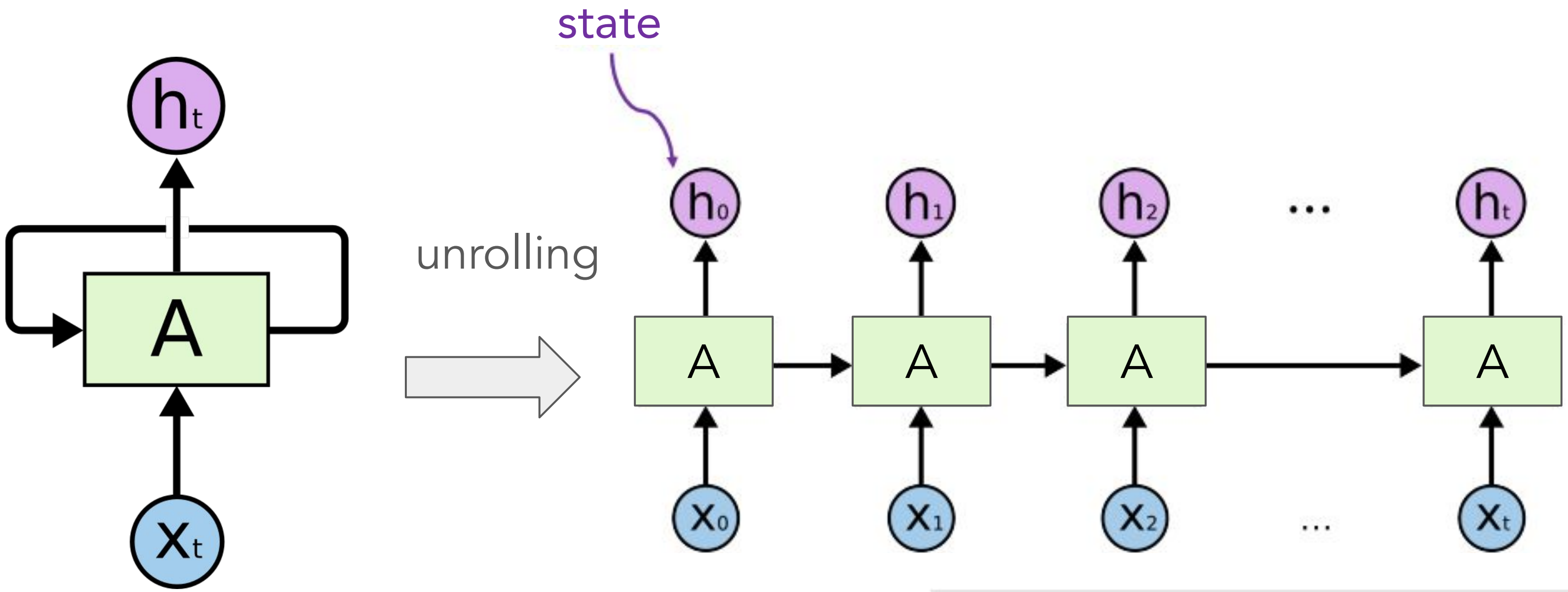

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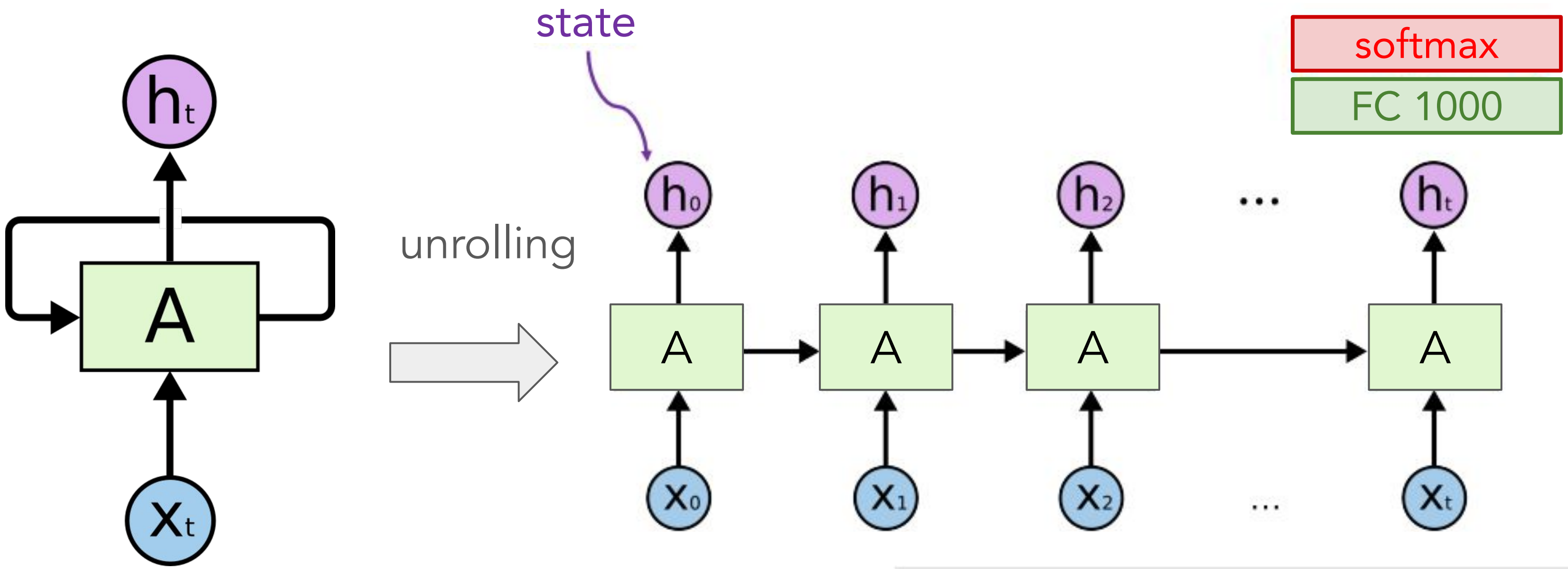

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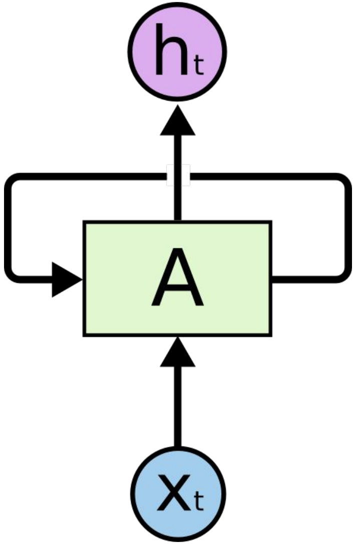
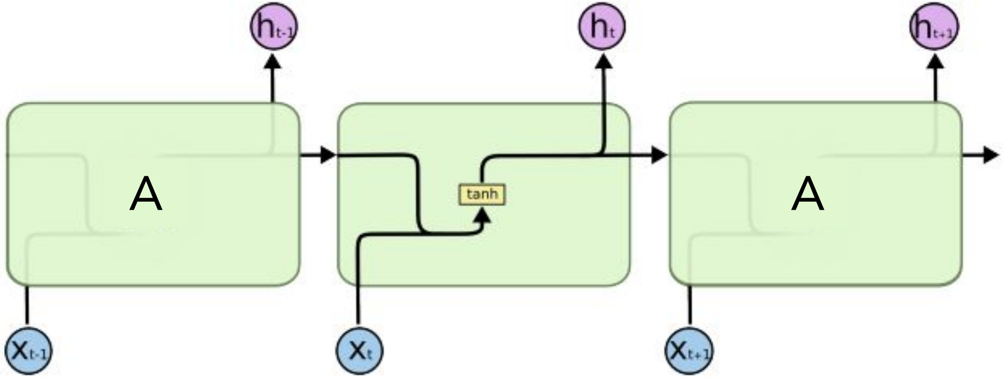
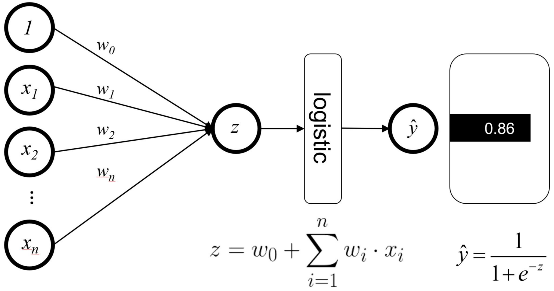
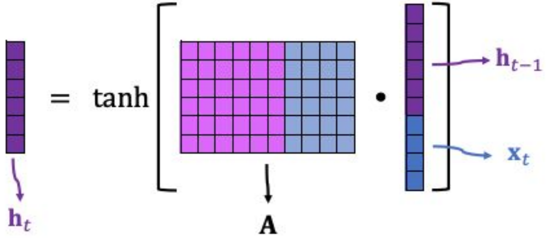
Simple RNN Cell









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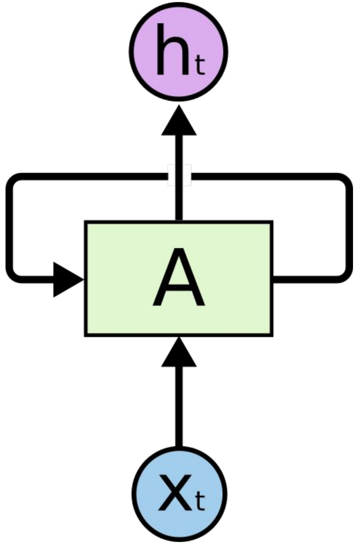
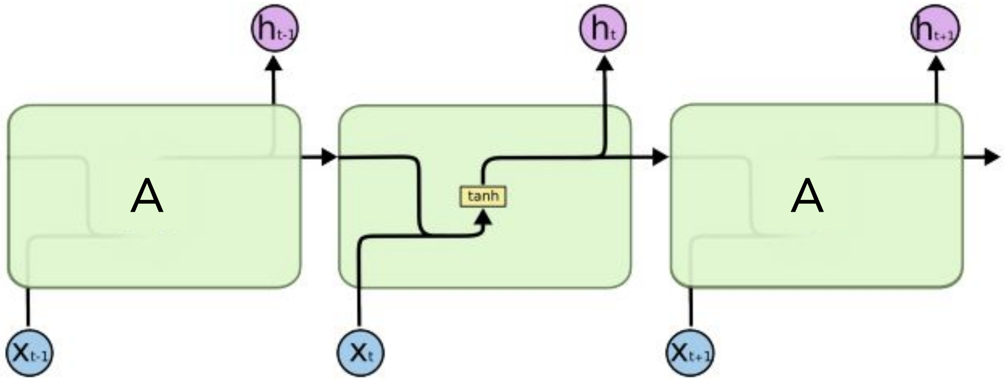
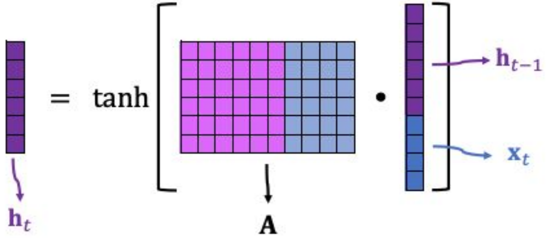
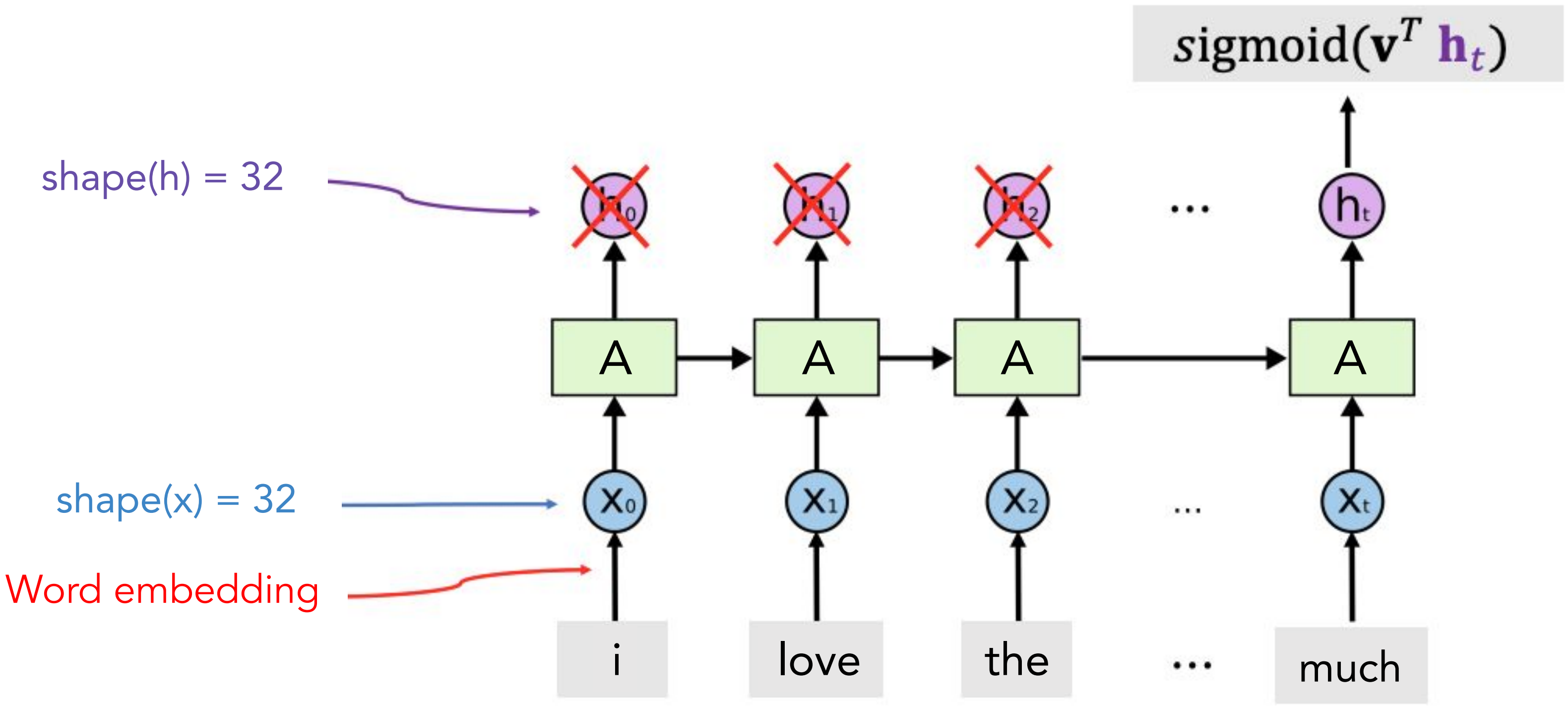




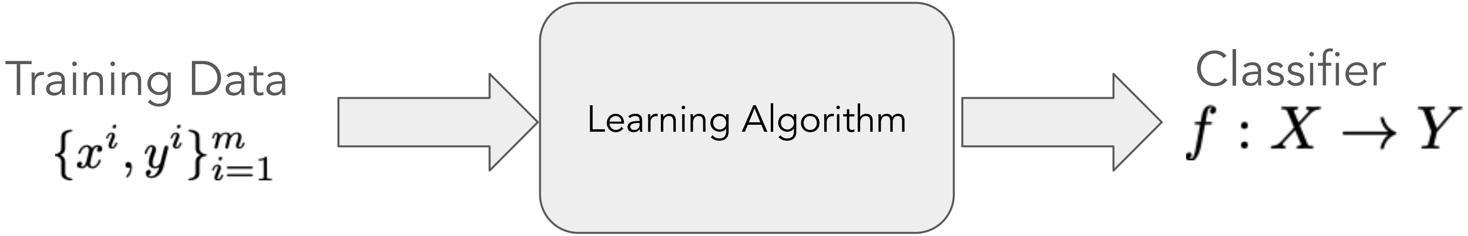
Binary step		$\begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$
Logistic, sigmoid, or soft step		$\sigma(x) \doteq \frac{1}{1+e^{-x}}$
Hyperbolic tangent (tanh)		$\tanh(x) \doteq \frac{e^x - e^{-x}}{e^x + e^{-x}}$
Rectified linear unit (ReLU)[13]		$(x)^+ \doteq \begin{cases} 0 & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases} = \max(0, x) = x\mathbf{1}_{x>0}$
Gaussian Error Linear Unit (GELU)[5]		$\frac{1}{2}x\left(1 + \text{erf}\left(\frac{x}{\sqrt{2}}\right)\right) = x\Phi(x)$ where erf is the gaussian error function.
Softplus[14]		$\ln(1 + e^x)$
Exponential linear unit (ELU)[15]		$\begin{cases} \alpha\left(e^x - 1\right) & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$ with parameter α



Simple RNN for IMDB Review



Sequential Multiclass Logistic Regression Algorithms



Multiclass Classification $Y \in \{0, 1, \ldots, k\}$

Multiclass Logistic Regression Pipeline

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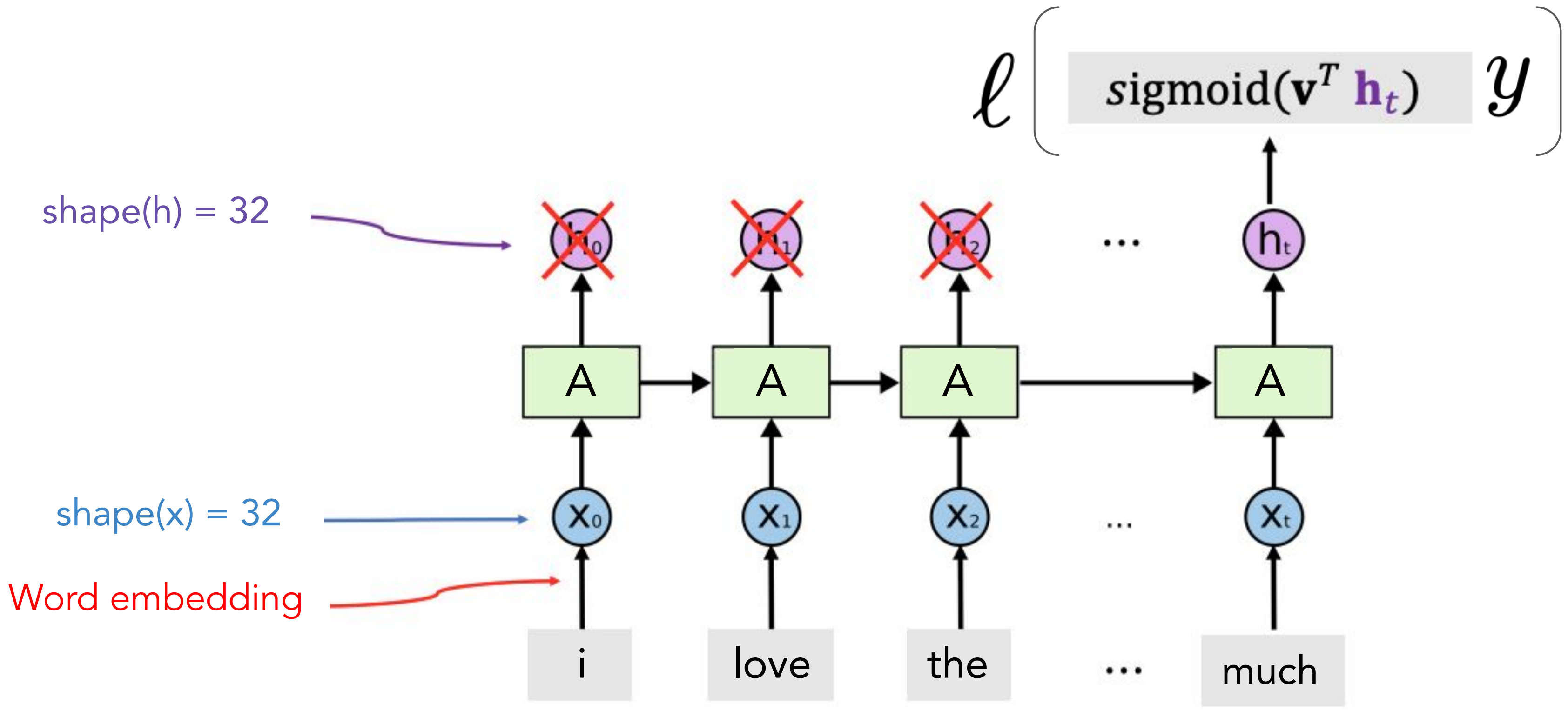
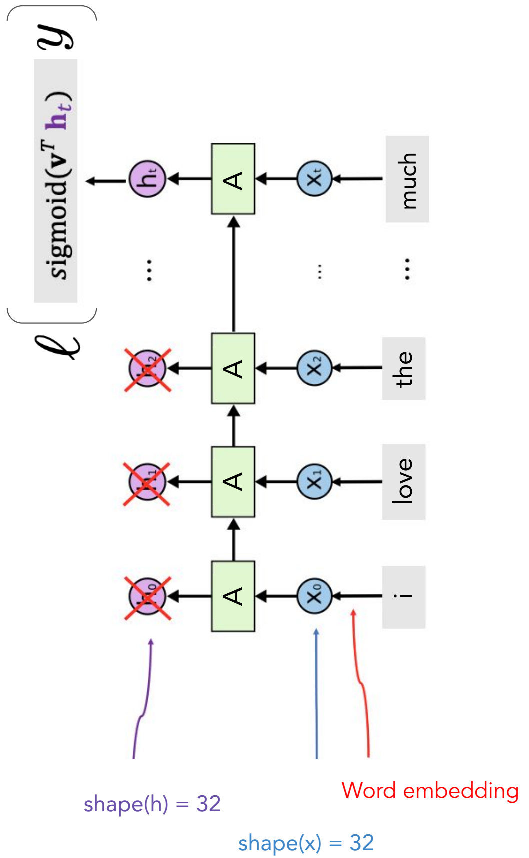
Backpropagation SGD



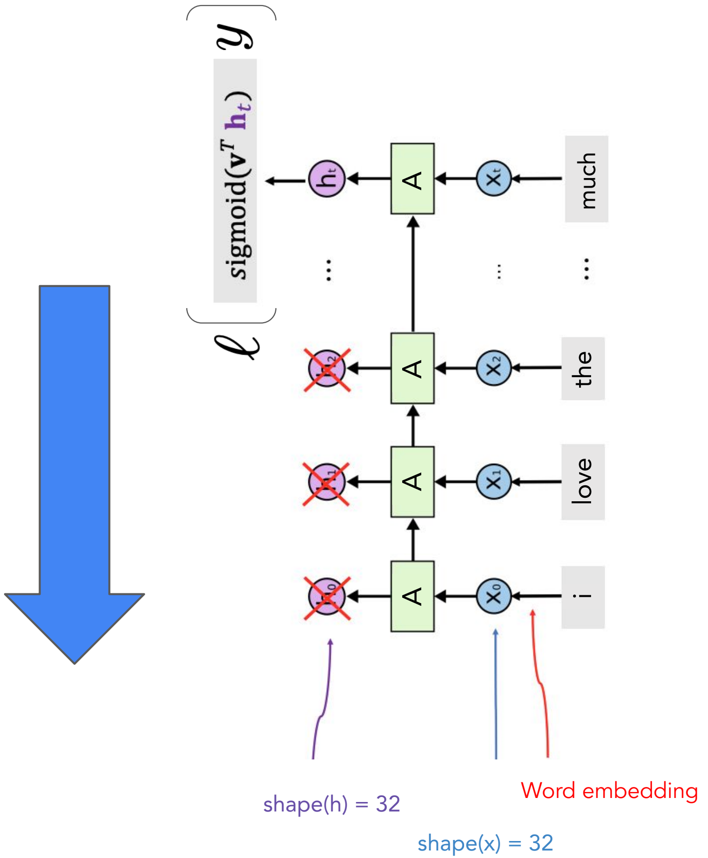
Backpropagation SGD

Layer ⟺ Time-step

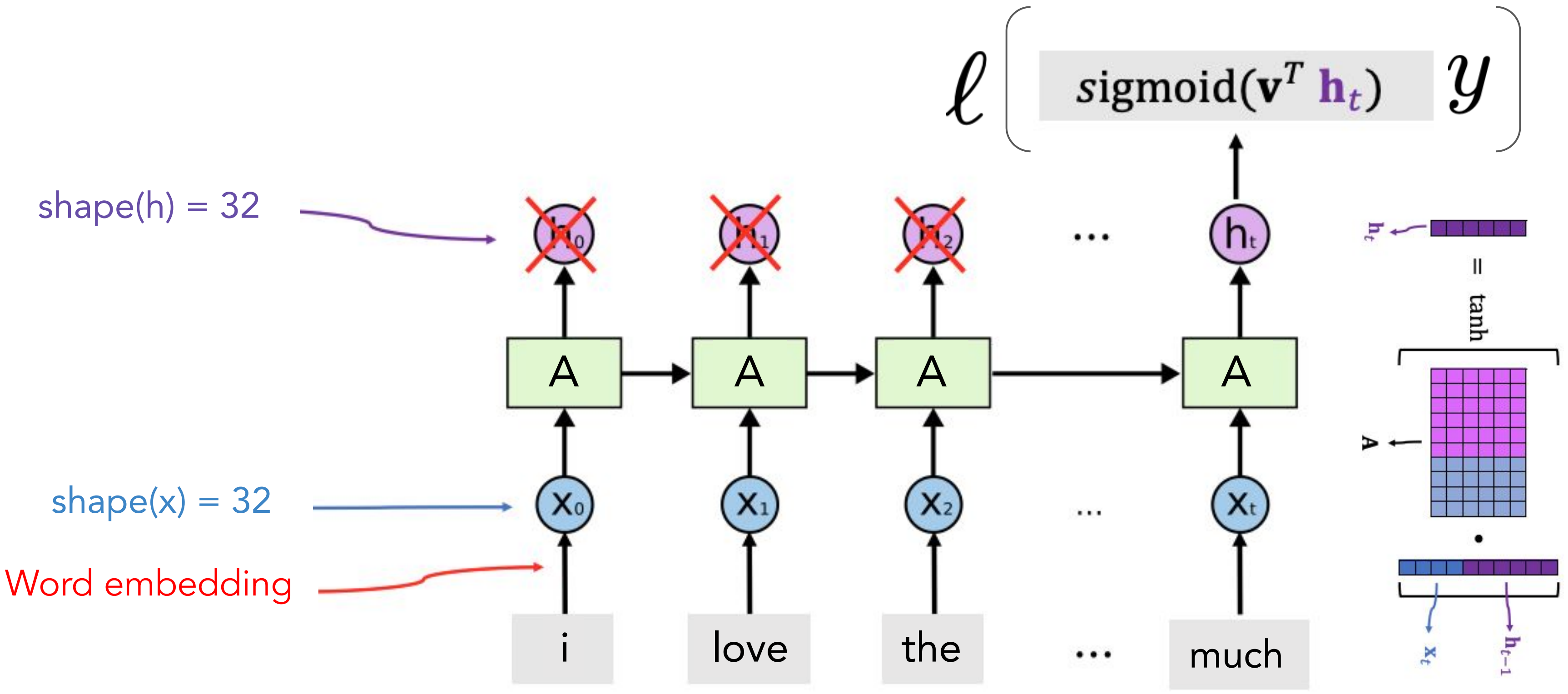


Backpropagation SGD

Backpropagation through Time (BPTT)

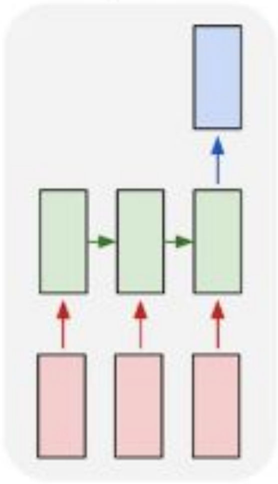


Backpropagation SGD



More Usages of RNNs

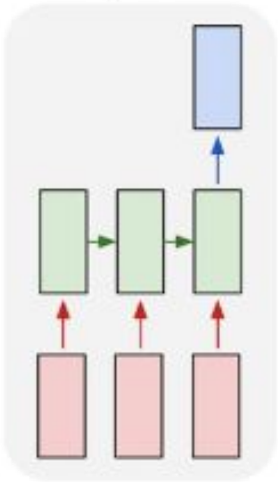
many to one



IMDB text review classification

More Usages of RNNs

many to one



one to many

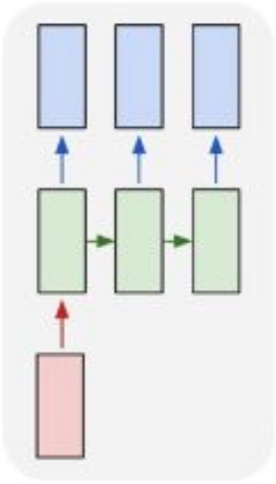
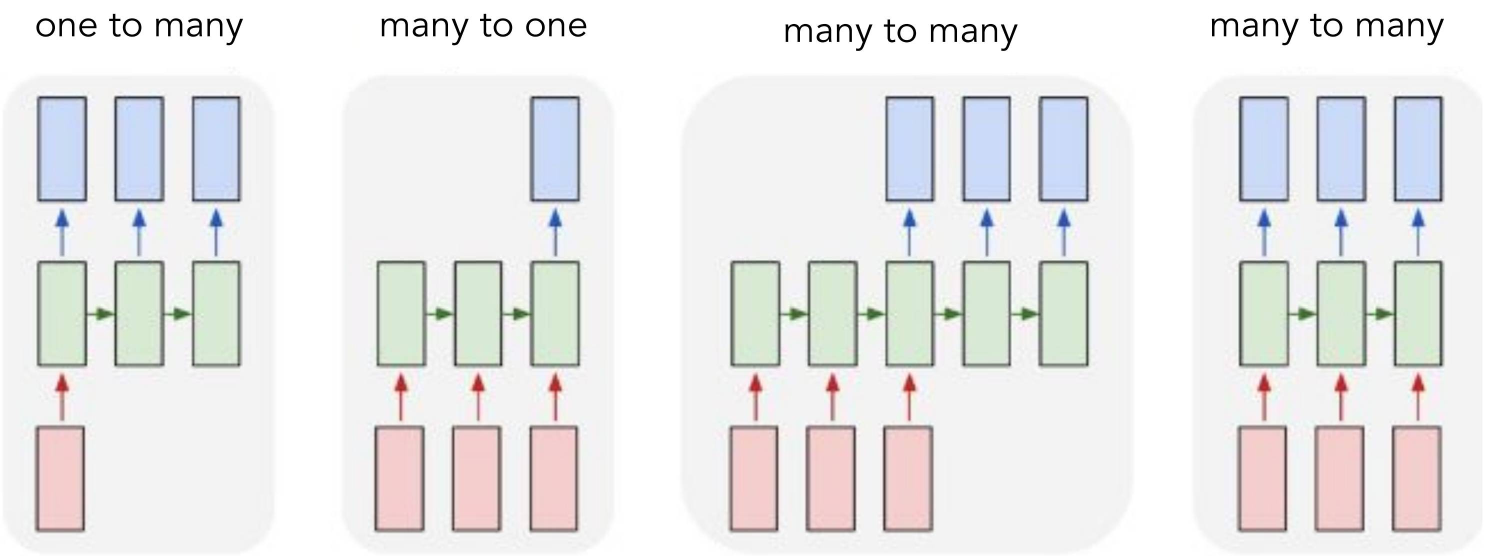


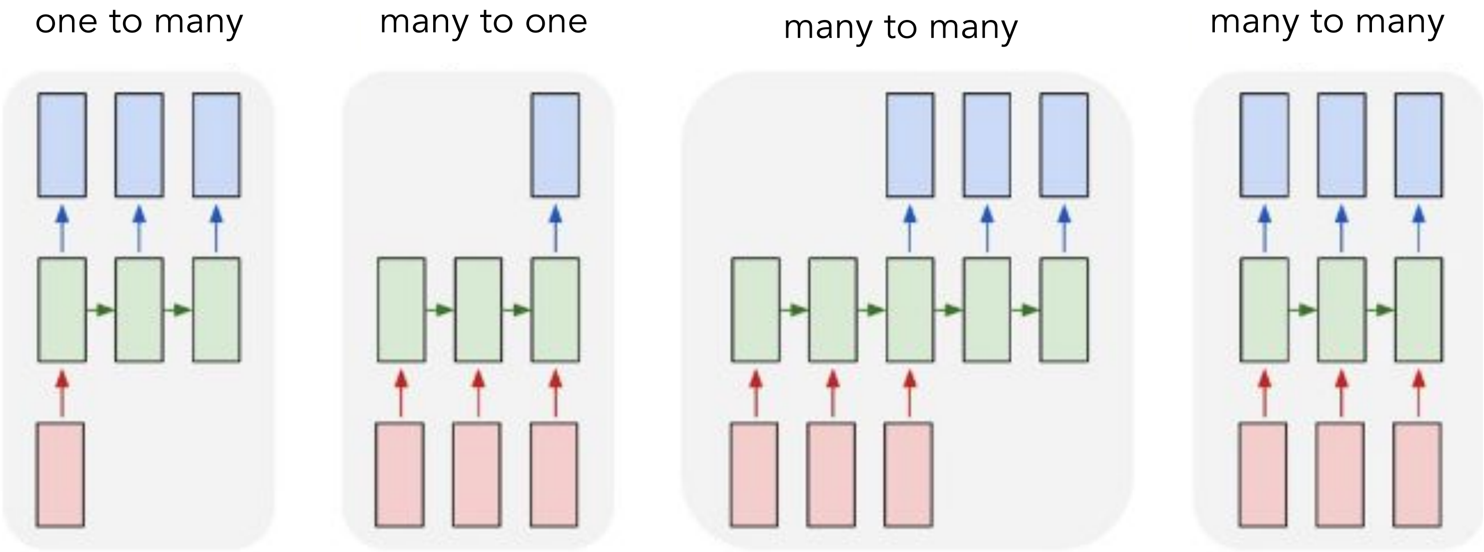
Image Captioning
image -> sequence of words

More Usages of RNNs



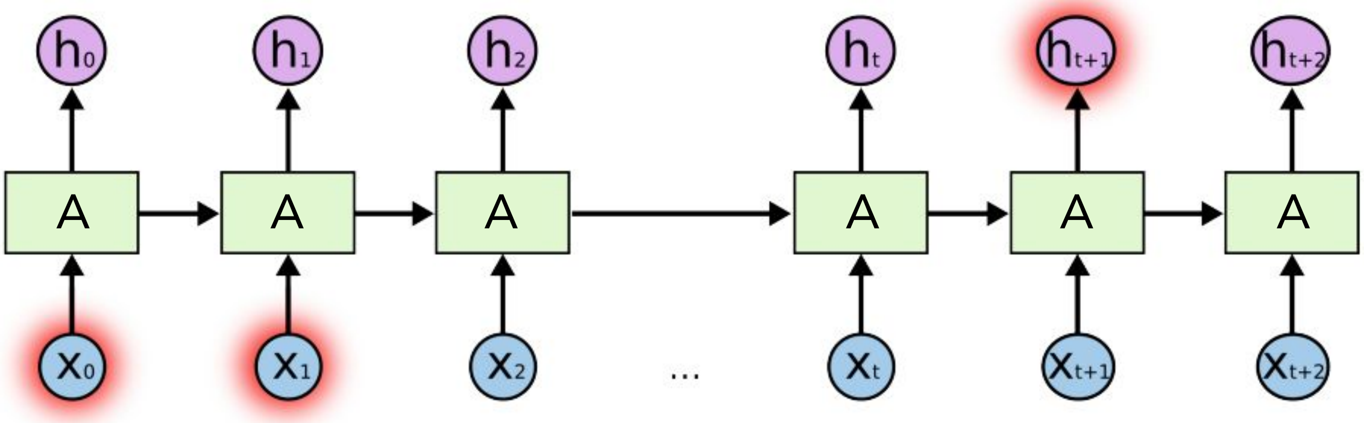
Translation

Training of RNNs



Backpropagation Through Unrolling Steps

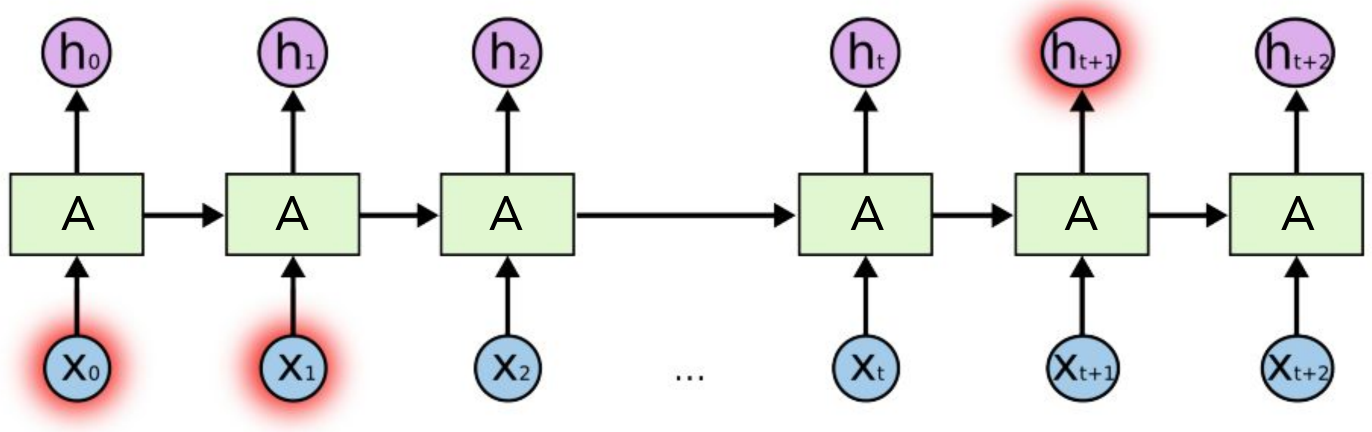
Simple RNN is not good at long-term dependence



$\mathbf{h}_{100}$ is almost irrelevant to $\mathbf{x}_1$: $\frac{\partial\, \mathbf{h}_{100}}{\partial\, \mathbf{x}_1}$ is near zero or exploding.

Gradient Vanishing Again!

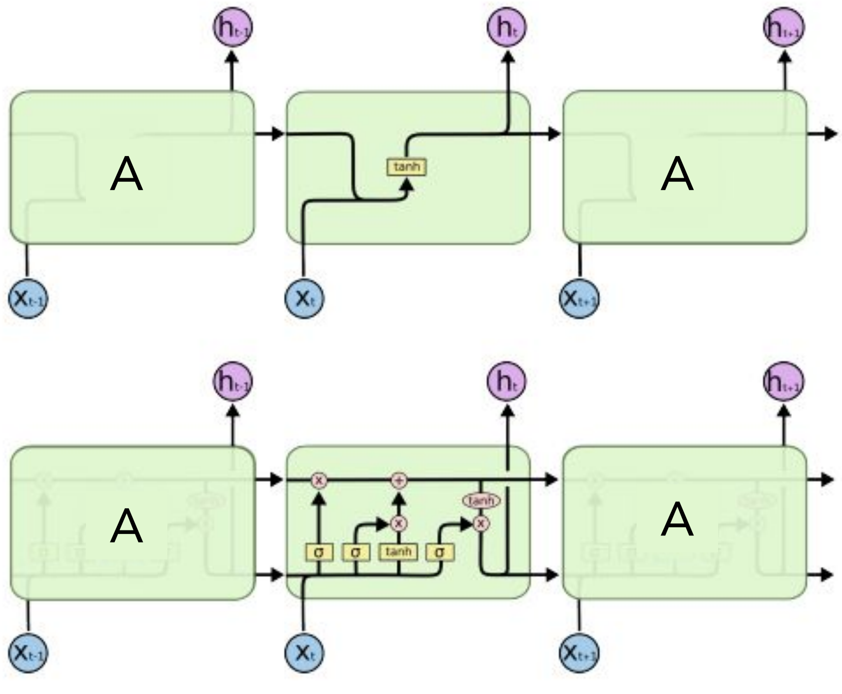
Simple RNN is not good at long-term dependence



$\mathbf{h}_{100}$ is almost irrelevant to $\mathbf{x}_1$: $\frac{\partial\, \mathbf{h}_{100}}{\partial\, \mathbf{x}_1}$ is near zero or exploding.

Gradient Vanishing Again! -> ShortCut connection

Long Short Term Memory (LSTM)



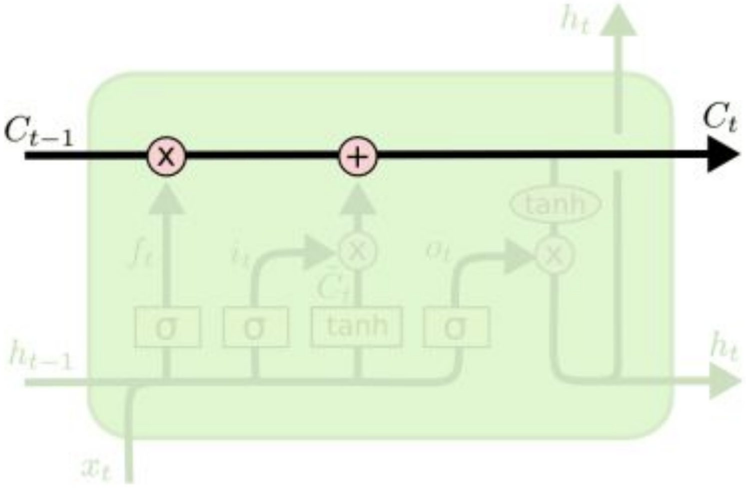
Simple RNN

LSTM

Figures from Christopher Olah’s blog

LSTM Cell: Conveyor Belt

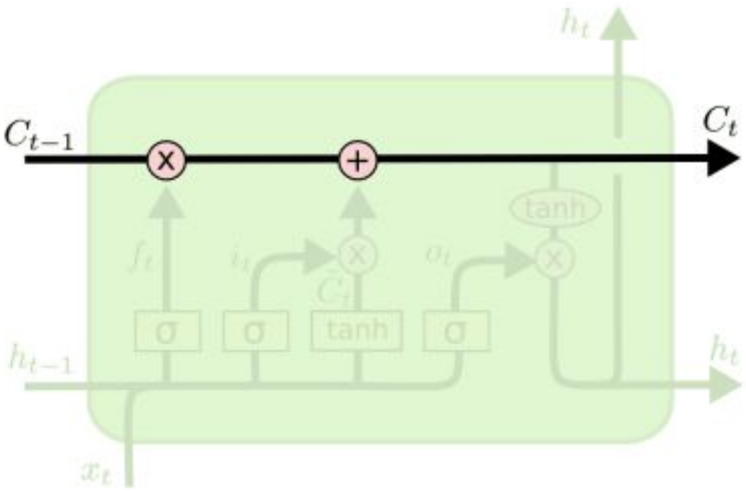
The past information directly flows to the future.



$$C_t = C_{t-1} \circ f_t + \hat{C}_t \circ i_t$$

LSTM Cell: Conveyor Belt

The past information directly flows to the future. (ShortCut connection)



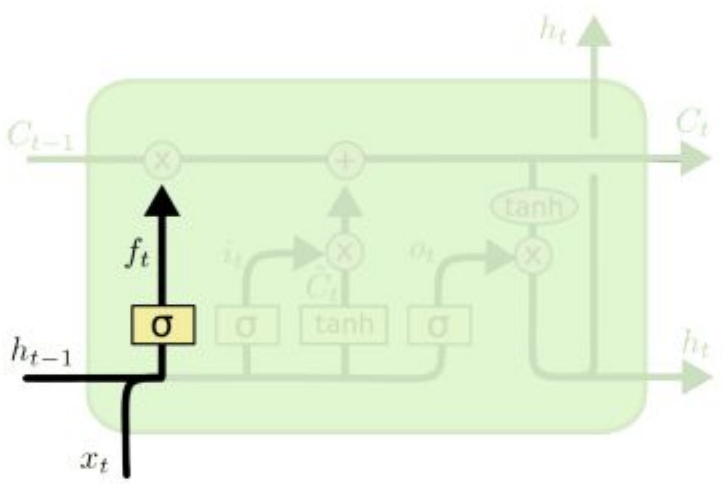
$$C_t = C_{t-1} \circ f_t + \hat{C}_t \circ i_t$$

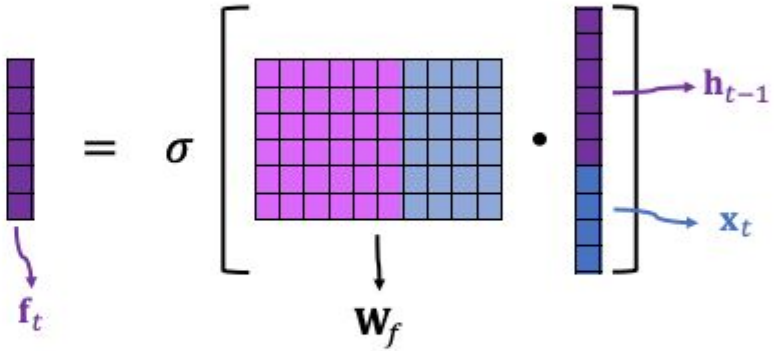
LSTM Cell: Forget Gate

A value of *zero* means "let *nothing* through."

A value of *one* means "let *everything* through!"

$$f_t = \sigma(W_f x_t + U_f h_{t-1})$$



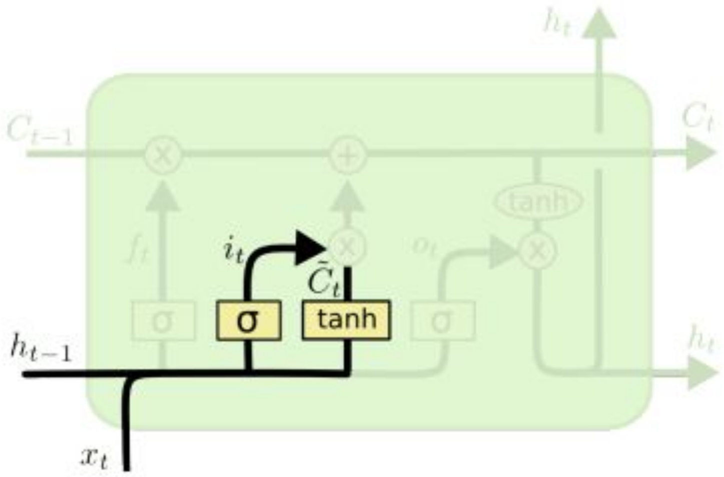


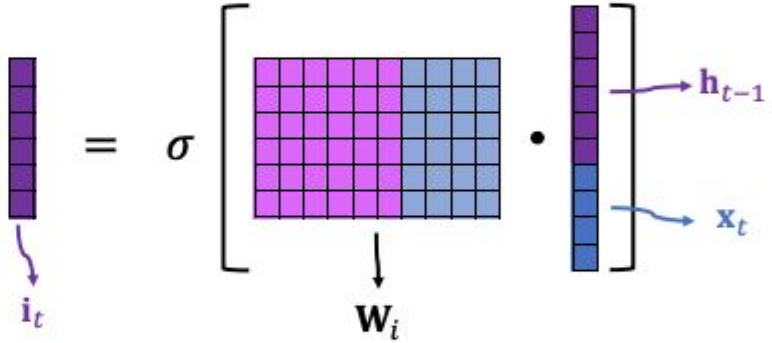
$$C_t = C_{t-1} \circ f_t + \hat{C}_t \circ i_t$$

LSTM Cell: Input Gate

How much information current context provided.

$$i_t = \sigma(W_i x_t + U_i h_{t-1})$$



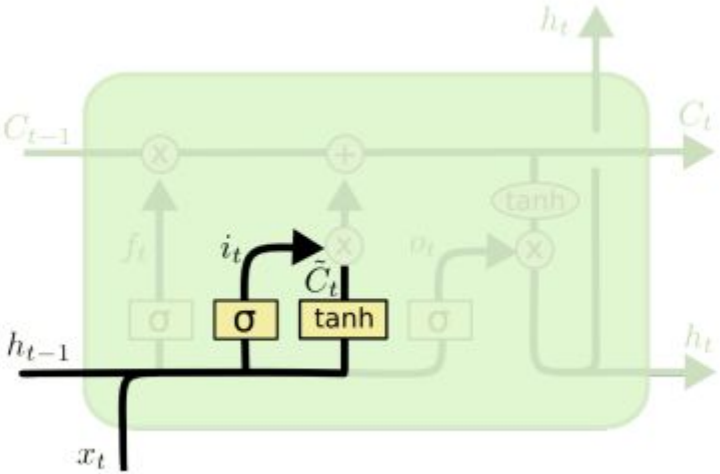


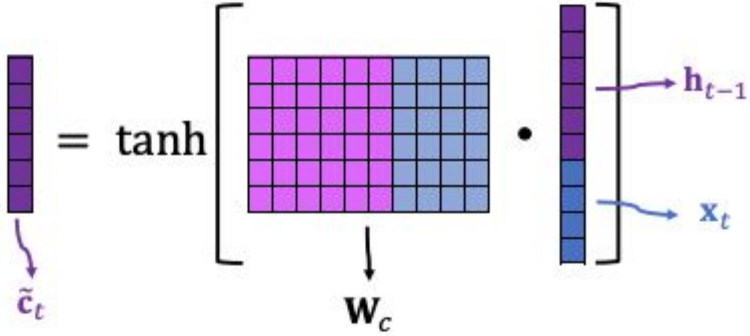
$$C_t = C_{t-1} \circ f_t + \hat{C}_t \circ i_t$$

LSTM Cell: New Value

"local" context, only up to immediately preceding state

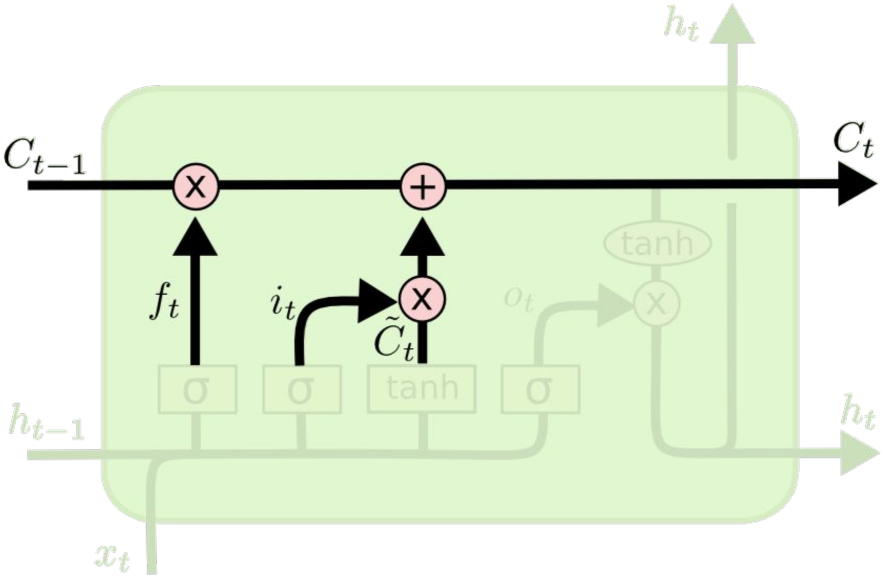
$$\hat{C}_t = \sigma(W_c x_t + U_c h_{t-1})$$





$$C_t = C_{t-1} \circ f_t + \hat{C}_t \circ i_t$$

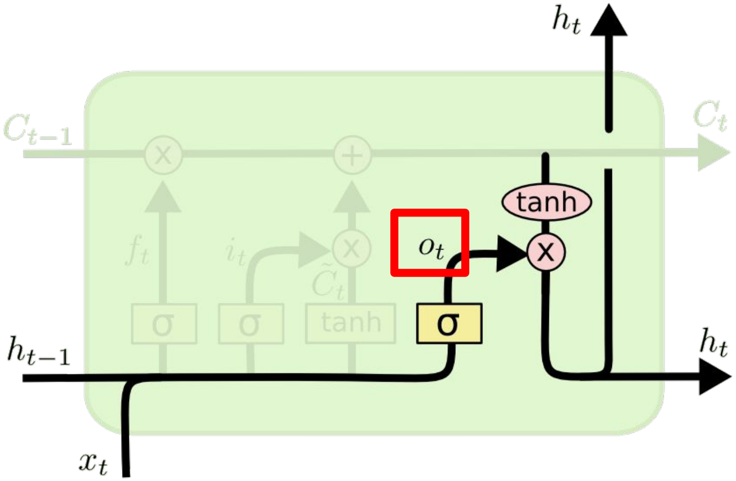
LSTM Cell: Update Conveyor Belt

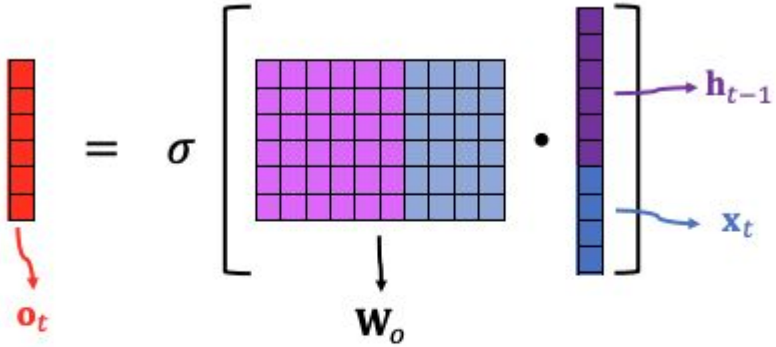


$$C_t = C_{t-1} \circ f_t + \hat{C}_t \circ i_t$$

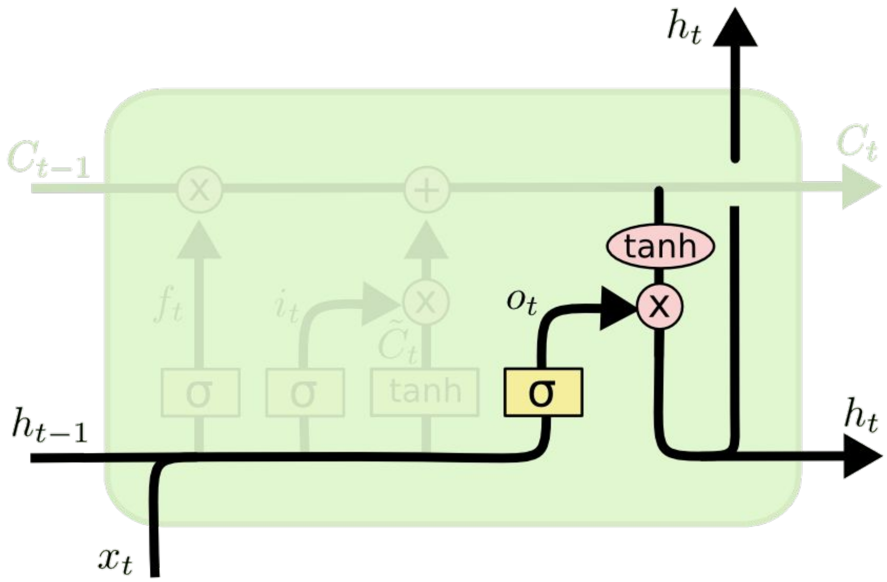
LSTM Cell: Output Gate

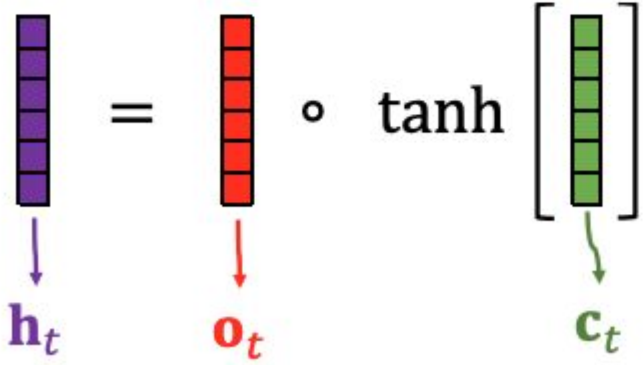
$$h_t = o_t * \tanh\left(C_t\right)$$





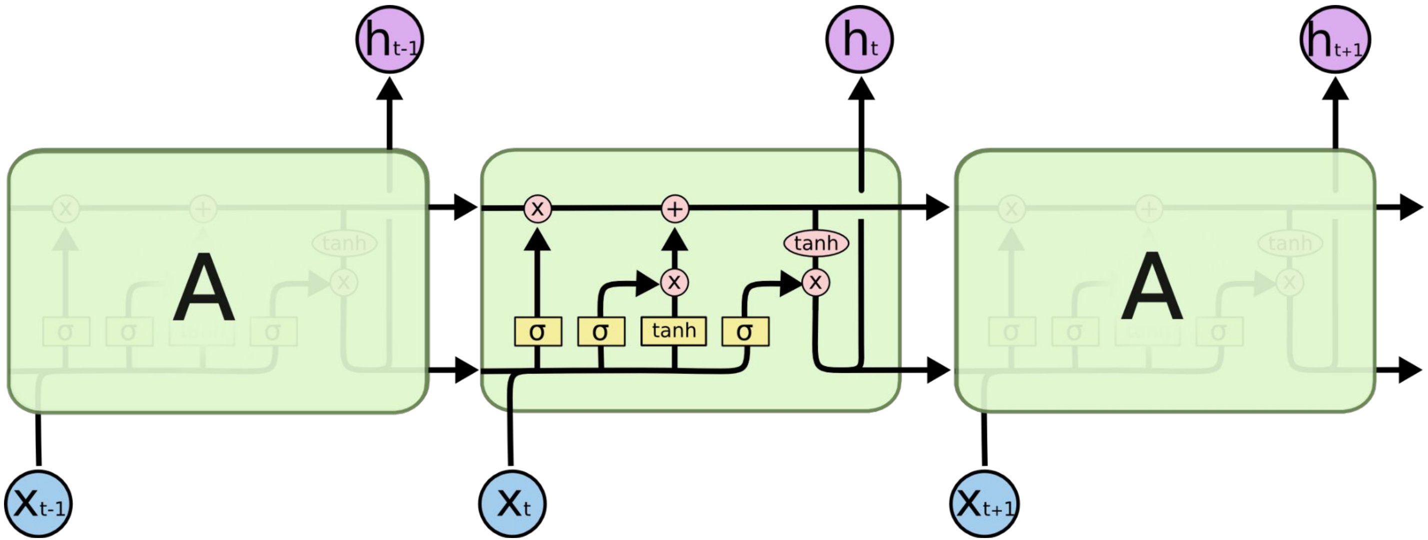
LSTM Cell: Update State





$$h_t = o_t * \tanh\left(C_t\right)$$

LSTM Cell



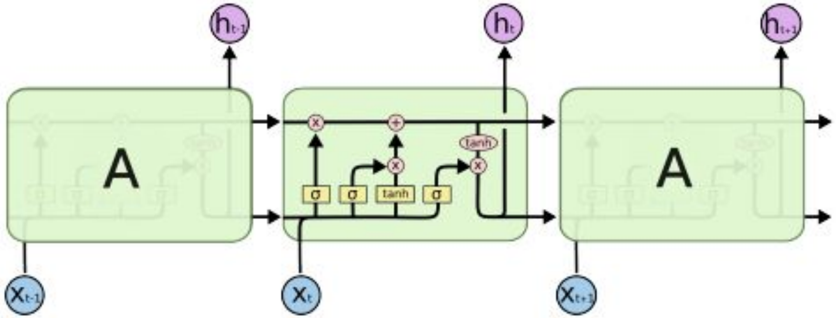
Auto-differentiation Packages

PyTorch

JAX

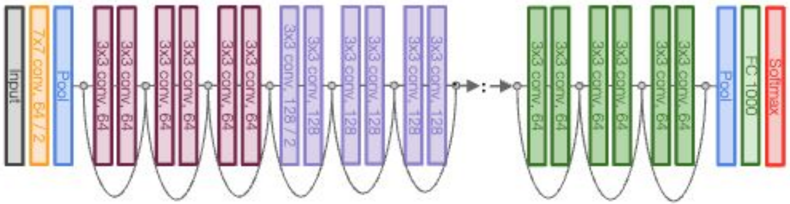
Tensorflow

LSTM vs. ResNet



$$C_t = C_{t-1} \circ f_t + \hat{C}_t \circ i_t$$

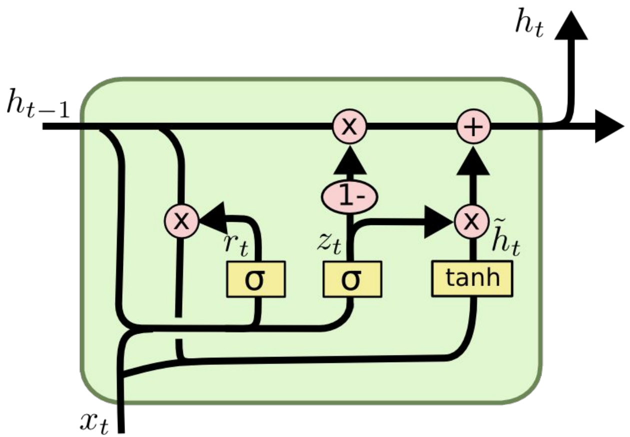
LSTM



Similar to ResNet

Other Variants of RNN

Gated Recurrent Unit



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

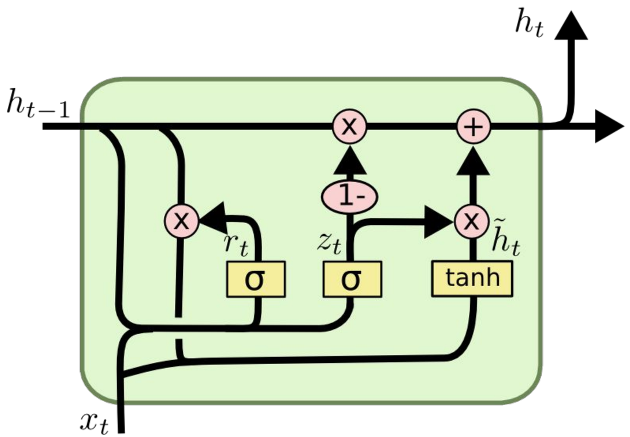
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

Other Variants of RNN

Gated Recurrent Unit



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$\boxed{h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t}$$

Summary

- RNNs allow a lot of flexibility in architecture design

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- Vanilla RNNs are simple but don't work very well

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- RNNs allow a lot of flexibility in architecture design
- Vanilla RNNs are simple but don't work very well
- Backward flow of gradients in RNN can explode or vanish. Exploding is controlled with gradient clipping. Vanishing is controlled with additive interactions (LSTM)

PyTorch session will be online next Monday

Q&A