Georgia Tech College of Computing

CX4240 Spring 2026

Latent Variable Model:

Variational Auto-Encoder

Bo Dai
School of CSE, Georgia Tech
bodai@cc.gatech.edu

Logistics

- Optional Proposal (10% bonus)
    - Introduction: Problem definition and motivation
    - Background & Related Work: Background info and literature survey (from today's view point)
- The deadline will be on March 13th.
- The practice exam has been released on March 6th on the course website.
- One exam will be held on March 18th in lieu of the regular class.
- We will have a makeup exam on March 17th in CODA C1315
    - You need an approval of absence to register this makeup exam
    - You may need the building access. If so, please gather at 2nd floor in CODA on 3:15pm and TA will lead you to the room.
    - Please register the makeup exam by March 16th (sending email to instructors and TAs).

Logistics

- Final Report (25%):
- All write-ups should use the NeurIPS style, up to 6 pages excluding references. It should have roughly the following format:
  - Introduction: Problem definition and motivation
  - Background & Related Work: Background info and literature survey (from today's view point)
  - Methods:
    - Details of the method reproduced
    - Intuition why the method is better (echoing the motivation: which special mechanism in the method will solve the motivated problem)
  - Experiments:
    - Description of your testbed and a list of questions your experiments are designed to answer
    - Details of the experiments and results
  - Conclusion: Discussion and future work (from today's view point, you may have some solutions already)
- The project final report will be due at 11:59 PM on May 4th.

Logistics

- 30% for proposed method (soundness)
- 30% for correctness, completeness, and difficulty of experiments and figures
- 20% for empirical and theoretical analysis of results and methods
- 20% for quality of writing (clarity, organization, flow, etc.)

We will have 4 presentation sessions (April 13th - April 22nd).

Presentation Signup from April 1st.

Each team will have 15mins for presentation

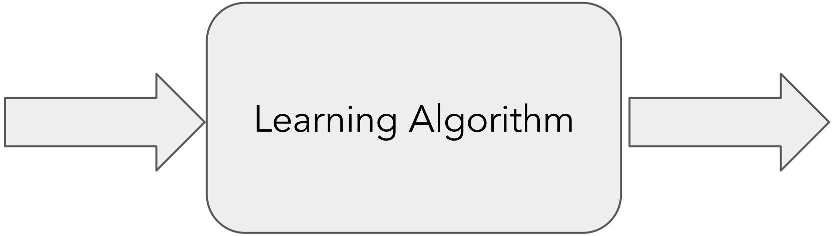
- 10% for presentation completed in ~~10mins~~ 12mins-13mins.
  - 2-3mins for questions and comments

Supervised Learning vs. Unsupervised Learning

Training Data

Supervised $\{x^i, y^i\}_{i=1}^{m}$

Unsupervised $\{x_i\}_{i=1}^{m}$



Target Function

$f : X \rightarrow Y$

Density Estimation $p(x)$

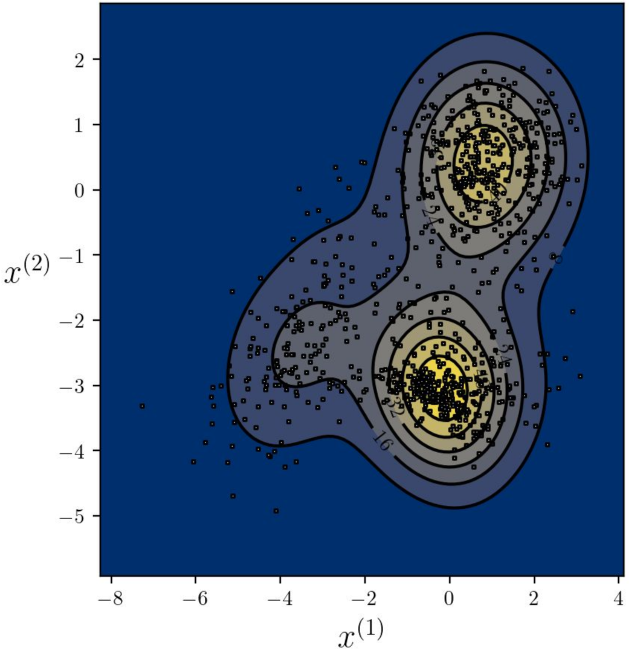
Clustering $f : X \rightarrow Y$

$Y \in \{0, 1, \ldots, k\}$

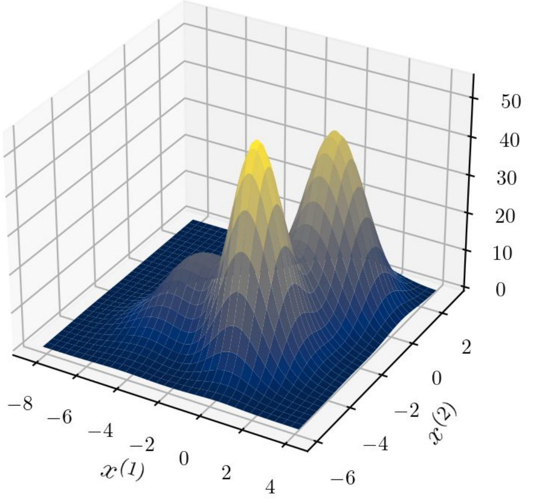
Dimension Reduction $f : X \rightarrow Y$

$X \in \mathbb{R}^p, \quad Y \in \mathbb{R}^d$

Density Estimation



$\{x_i\}_{i=1}^{m}$

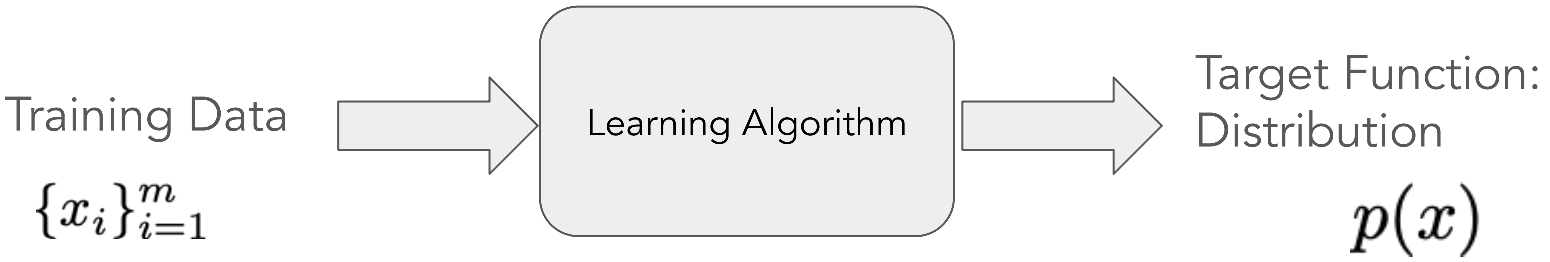


$p(x)$

Generative Models

$x \sim p(x)$

Density Estimation: Gaussian Mixture Model



Density Estimation Pipeline

1. Build probabilistic models
   Gaussian Mixture Model
2. Derive loss function (by MLE or MAP....)
   MLE
3. Select optimizer
   EM

Gaussian Mixture Model

Class mixture prior: $P(y)$ $\quad \pi = (\pi_1, \pi_2, \ldots, \pi_k), \quad \sum_{i=1}^{k} \pi_i = 1, \pi_i \geq 0$

Class conditional distribution: $p(x|y) = \mathcal{N}(x|\mu_y, \Sigma_y)$

Marginal distribution: $P(x) = \sum_y P(x|y)P(y) = \sum_{i=1}^{k} \pi_i \mathcal{N}(x|\mu_i, \Sigma_i)$

GMM

$$\max_{\{\pi_i,\mu_i,\Sigma_i\}_{i=1}^k} \sum_{j=1}^{m} \log p(x_j) = \sum_{j=1}^{m} \log \Big(\sum_{i=1}^{k} p(x_j, y_j = i) \Big) = \sum_{j=1}^{m} \log \Big(\sum_{i=1}^{k} \pi_i \mathcal{N}(x_j | \mu_i, \Sigma_i) \Big)$$

Want $\arg\max_{\theta} \log L(\theta)$ subject to $\sum_{j=1}^{k} \pi_j = 1$

GMM -> Latent Variable Models

$$\max_{\{\pi_i,\mu_i,\Sigma_i\}_{i=1}^k} \sum_{j=1}^m \log p(x_j) = \sum_{j=1}^m \log\Big(\underbrace{\sum_{i=1}^k p(x_j, y_j = i)}\Big) = \sum_{j=1}^m \log\Big(\sum_{i=1}^k \pi_i \mathcal{N}(x_j|\mu_i, \Sigma_i)\Big)$$

marginalize latent variables

Want $\arg\max_\theta \log L(\theta)$ subject to $\sum_{j=1}^k \pi_j = 1$

Expectation-Maximization

For t = 1……

- E-Step: Guess sample labels based on current model

$$\tau_j^l = \frac{\pi_l \mathcal{N}(x_j|\mu_l, \Sigma_l)}{\sum_{l=1}^{k} \pi_l \mathcal{N}(x_j|\mu_l, \Sigma_l)}$$

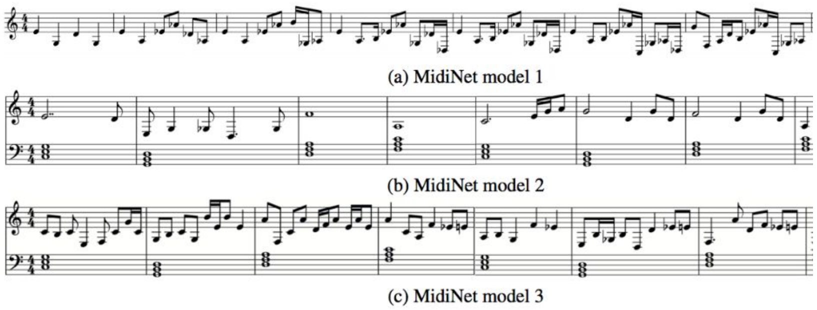
- M-Step: Update the parameters with current labels (Gaussian-Naive Bayes)

$$\mu_k = \frac{\sum_{i=1}^{m} \tau_k^i x^i}{\sum_{i=1}^{m} \tau_k^i}, \quad \pi_k = \frac{\sum_{i=1}^{m} \tau_k^i}{m}, \quad \Sigma_k = \frac{\sum_{i=1}^{m} \tau_k^i (x^i - \mu_k)(x^i - \mu_k)^\top}{\sum_{i=1}^{m} \tau_k^i}$$

This procedure is actually minimizing an upper bound of negative MLE, therefore, it converges

Density Estimation: (Deep) Generative Models

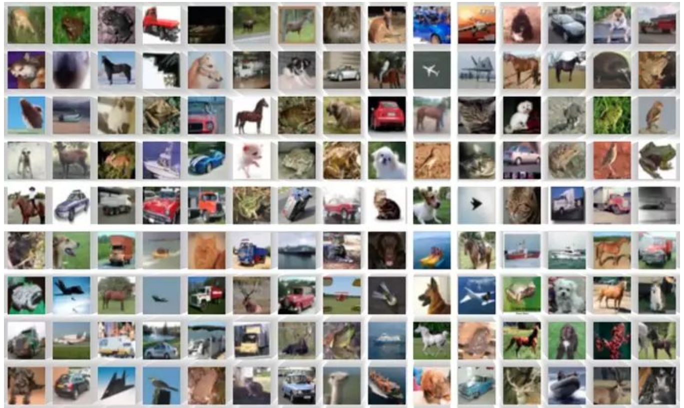
$$x \sim p(x)$$



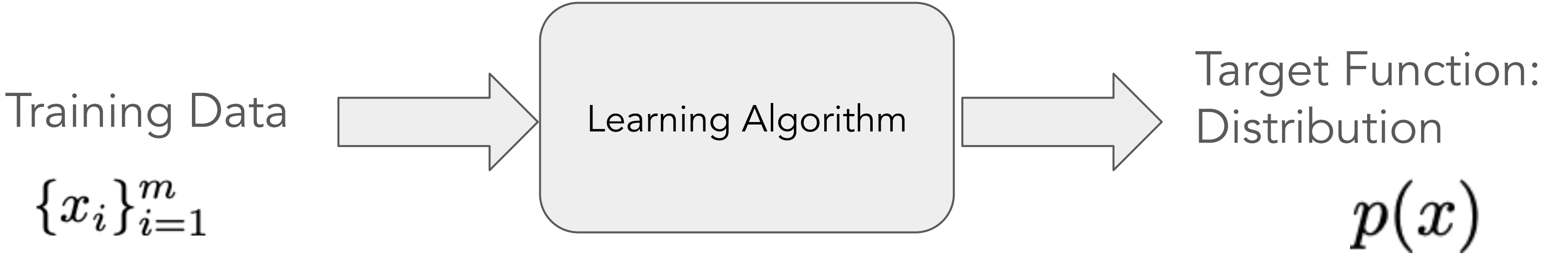
(a) MidiNet model 1

(b) MidiNet model 2

(c) MidiNet model 3



Generative Model: (Deep) Latent Variable Models



Density Estimation Pipeline

1. Build probabilistic models
   Deep Latent Variable Model
2. Derive loss function (by MLE or MAP….)
3. Select optimizer

Latent Variable Models

GMMs $\sum_{i=1}^{k} \pi_i \mathcal{N}(x|\mu_i, \Sigma_i)$

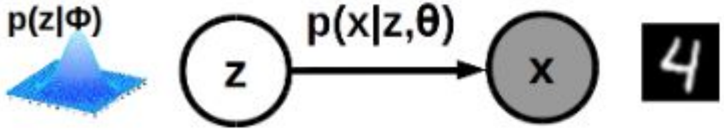
Latent Variable Models

Figure 5: 1024 × 1024 images generated using the CELEBA-HQ dataset. See Appendix F for a larger set of results, and the accompanying video for latent space interpolations.

GMMs $$\sum_{i=1}^{k} \pi_i \mathcal{N}(x|\mu_i, \Sigma_i)$$

Not flexible enough

Latent Variable Models



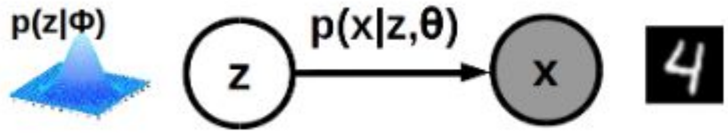
$$p(x) = \int p(x|z)p(z)dz$$

Figure 5: 1024 × 1024 images generated using the CELEBA-HQ dataset. See Appendix F for a larger set of results, and the accompanying video for latent space interpolations.

GMMs $\sum_{i=1}^{k} \pi_i \mathcal{N}(x|\mu_i, \Sigma_i)$

Not flexible enough

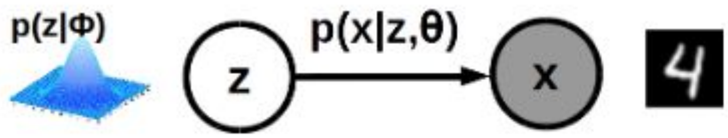
Latent Variable Models



$$p(x) = \int p(x|z)p(z)dz$$

Infinite-many components: continuous z

Latent Variable Models



$$p(x) = \int p(x|z)p(z)dz$$

Infinite-many components: continuous z

Make $p(x|z)$ more flexible: deep models

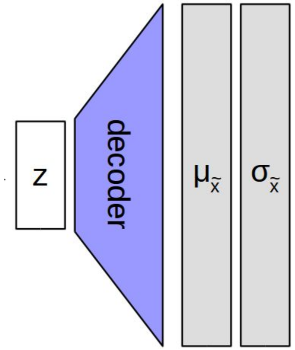
Deep Gaussian Distribution

Gaussian Distribution

$$p(\mathbf{x} \mid \mathbf{z}) = \mathcal{N}(\mathbf{x} \mid \boldsymbol{\mu}_z, \boldsymbol{\Sigma}_z) = \frac{1}{\sqrt{(2\pi)^d \det(\boldsymbol{\Sigma}_z)}} \exp\Big(-\tfrac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_z)^T \boldsymbol{\Sigma}_z^{-1} (\mathbf{x} - \boldsymbol{\mu}_z)\Big)$$

Deep Gaussian Distribution

$$p(\mathbf{x} \mid \mathbf{z}) = \mathcal{N}(\mathbf{x} \mid \boldsymbol{\mu}(z), \boldsymbol{\Sigma}(z)) = \frac{1}{\sqrt{(2\pi)^d \det(\boldsymbol{\Sigma}(z))}} \exp\Big(-\tfrac{1}{2}(\mathbf{x} - \boldsymbol{\mu}(z))^T \boldsymbol{\Sigma}(z)^{-1} (\mathbf{x} - \boldsymbol{\mu}(z))\Big)$$



$$p(\mathbf{x} \mid \mathbf{z}) = \mathcal{N}\Big(\mathbf{x} \,\Big|\, \mu_{\tilde{x}}(z),\ \operatorname{diag}\big(\sigma^2_{\tilde{x}}(z)\big)\Big).$$

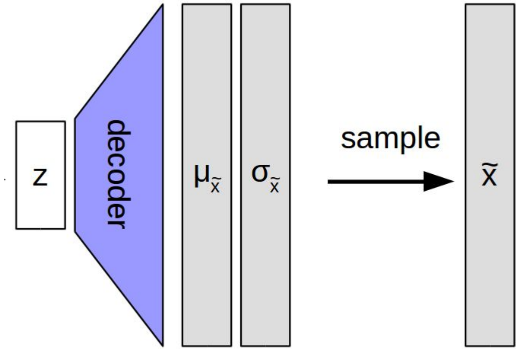
Deep Gaussian Distribution

Gaussian Distribution

$$p(\mathbf{x} \mid \mathbf{z}) = \mathcal{N}(\mathbf{x} \mid \boldsymbol{\mu}_z, \boldsymbol{\Sigma}_z) = \frac{1}{\sqrt{(2\pi)^d \det(\boldsymbol{\Sigma}_z)}} \exp\Big(-\tfrac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_z)^T \boldsymbol{\Sigma}_z^{-1} (\mathbf{x} - \boldsymbol{\mu}_z)\Big)$$

Deep Gaussian Distribution

$$p(\mathbf{x} \mid \mathbf{z}) = \mathcal{N}(\mathbf{x} \mid \boldsymbol{\mu}(z), \boldsymbol{\Sigma}(z)) = \frac{1}{\sqrt{(2\pi)^d \det(\boldsymbol{\Sigma}(z))}} \exp\Big(-\tfrac{1}{2}(\mathbf{x} - \boldsymbol{\mu}(z))^T \boldsymbol{\Sigma}(z)^{-1} (\mathbf{x} - \boldsymbol{\mu}(z))\Big)$$



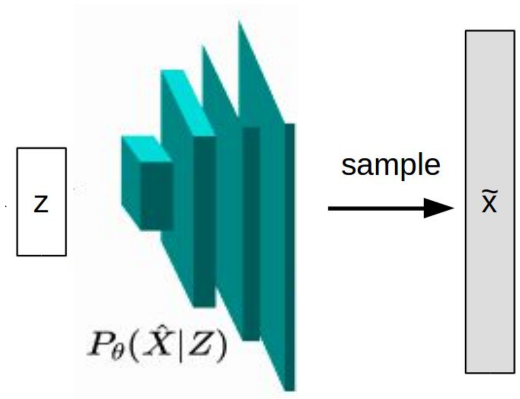
Deep Gaussian Distribution

Gaussian Distribution

$$p(\mathbf{x} \mid \mathbf{z}) = \mathcal{N}(\mathbf{x} \mid \boldsymbol{\mu}_z, \boldsymbol{\Sigma}_z) = \frac{1}{\sqrt{(2\pi)^d \det(\boldsymbol{\Sigma}_z)}} \exp\left(-\tfrac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_z)^T \boldsymbol{\Sigma}_z^{-1} (\mathbf{x} - \boldsymbol{\mu}_z)\right)$$

Deep Gaussian Distribution

$$p(\mathbf{x} \mid \mathbf{z}) = \mathcal{N}(\mathbf{x} \mid \boldsymbol{\mu}(z), \boldsymbol{\Sigma}(z)) = \frac{1}{\sqrt{(2\pi)^d \det(\boldsymbol{\Sigma}(z))}} \exp\left(-\tfrac{1}{2}(\mathbf{x} - \boldsymbol{\mu}(z))^T \boldsymbol{\Sigma}(z)^{-1} (\mathbf{x} - \boldsymbol{\mu}(z))\right)$$



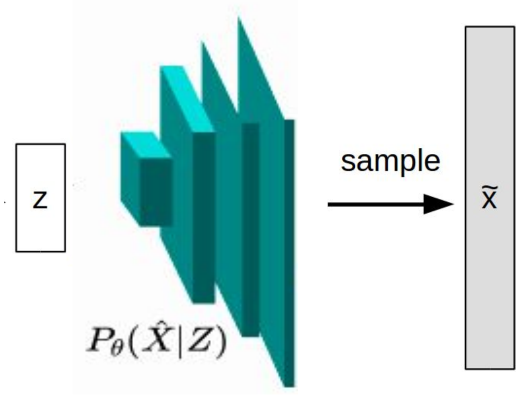
Deep Gaussian Distribution

Gaussian Distribution

$$p(\mathbf{x} \mid \mathbf{z}) = \mathcal{N}(\mathbf{x} \mid \boldsymbol{\mu}_z, \boldsymbol{\Sigma}_z) = \frac{1}{\sqrt{(2\pi)^d \det(\boldsymbol{\Sigma}_z)}} \exp\left(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_z)^T \boldsymbol{\Sigma}_z^{-1} (\mathbf{x} - \boldsymbol{\mu}_z)\right)$$

Deep Gaussian Distribution

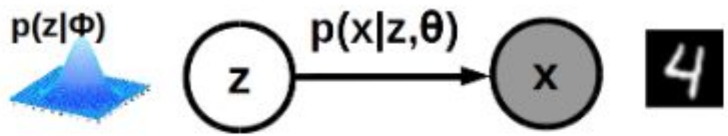
$$p(\mathbf{x} \mid \mathbf{z}) = \mathcal{N}(\mathbf{x} \mid \boldsymbol{\mu}(z), \boldsymbol{\Sigma}(z)) = \frac{1}{\sqrt{(2\pi)^d \det(\boldsymbol{\Sigma}(z))}} \exp\left(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}(z))^T \boldsymbol{\Sigma}(z)^{-1} (\mathbf{x} - \boldsymbol{\mu}(z))\right)$$



deep conv neural network

$$\mu_{W_\mu}(z), \sigma_{W_\sigma}(z)$$

Deep Latent Variable Models: Deep Gaussian LVM



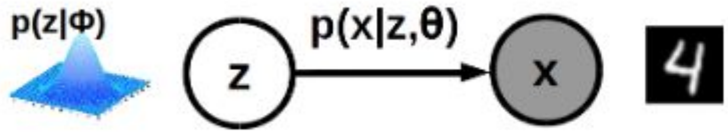
$$x \sim p(x) = \int p(x|z)p(z)dz$$

$$p(z) = \mathcal{N}(0, \sigma I)$$

$$\begin{aligned} p(\mathbf{x} \mid \mathbf{z}) &= \mathcal{N}(\mathbf{x} \mid \boldsymbol{\mu}(z), \boldsymbol{\Sigma}(z)) \\ &= \frac{1}{\sqrt{(2\pi)^d \det(\boldsymbol{\Sigma}(z))}} \exp\Big(-\tfrac{1}{2}(\mathbf{x} - \boldsymbol{\mu}(z))^T \boldsymbol{\Sigma}(z)^{-1} (\mathbf{x} - \boldsymbol{\mu}(z))\Big) \end{aligned}$$

Model Parameters $\quad \mu_{W_\mu}(z), \sigma_{W_\sigma}(z) \;\; \sigma$

Sampling as Generation

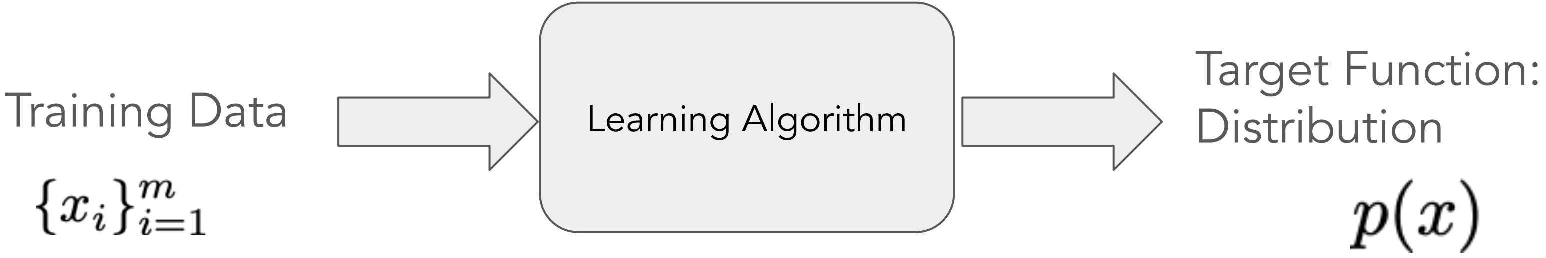


$$x \sim p(x) = \int p(x|z)p(z)dz$$

$$z \sim p(z) = \mathcal{N}(0, \sigma I)$$

$$x|z \sim \mathcal{N}(\mu(z), \sigma(z)I)$$

Generative Model: Latent Variable Models



Density Estimation Pipeline

1. Build probabilistic models
2. Derive loss function (by MLE or MAP….)
   MLE?
3. Select optimizer

MLE of Deep LVM

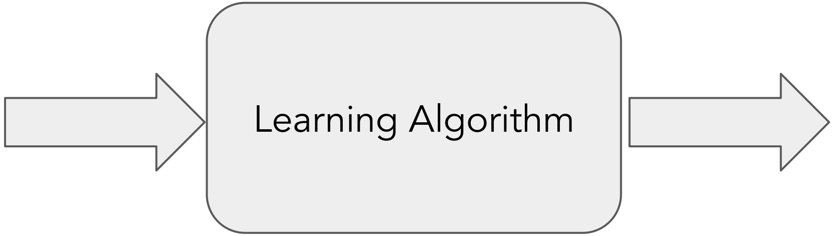
$$p(x) = \int p(x|z)p(z)dz$$

$$\max_{\sigma, W_\mu, W_\sigma} \sum_{i=1}^{m} \log p(x^i) = \sum_{i=1}^{m} \log \boxed{\int p(z)p(x^i|z)dz}$$

Generative Model: Latent Variable Models

Training Data

$\{x_i\}_{i=1}^{m}$



Target Function: Distribution

$p(x)$

Density Estimation Pipeline

1. Build probabilistic models
2. Derive loss function (by MLE or MAP….)
   MLE Approximation: Evidence Lower BOund (ELBO) of MLE
3. Select optimizer

Recall GMMs

$$\sum_{i=1}^{k} \pi_i \mathcal{N}(x|\mu_i, \Sigma_i)$$

- E-Step:

$$\tau_j^l = \frac{\pi_l \mathcal{N}(x_j|\mu_l, \Sigma_l)}{\sum_{l=1}^{k} \pi_l \mathcal{N}(x_j|\mu_l, \Sigma_l)}$$

- M-Step:

$$\max_{\pi_z, \mu_z, \Sigma_z} \sum_{i=1}^{m} \sum_{j=1}^{k} \tau_j^i \log \pi_j - \sum_{i=1}^{m} \log Z - \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{k} \tau_j^i (x^i - \mu_j)^\top \Sigma_j^{-1} (x^i - \mu_j)$$

log-joint probability

Recall GMMs

$$\sum_{i=1}^{k} \pi_i \mathcal{N}(x|\mu_i, \Sigma_i)$$

- E-Step:

$$\tau_j^l = \frac{\pi_l \mathcal{N}(x_j|\mu_l, \Sigma_l)}{\sum_{l=1}^{k} \pi_l \mathcal{N}(x_j|\mu_l, \Sigma_l)} = q(y^i = j|x^i)$$

- M-Step:

$$\max_{\pi_z, \mu_z, \Sigma_z} \sum_{i=1}^{m} \sum_{j=1}^{k} \tau_j^i \log \pi_j - \sum_{i=1}^{m} \log Z - \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{k} \tau_j^i (x^i - \mu_j)^\top \Sigma_j^{-1} (x^i - \mu_j)$$

$$\max_{\pi_z, \mu_z, \Sigma_z} \sum_{i=1}^{m} \sum_{j=1}^{k} q(y^i = j|x^i) \log p(x^i, y^i)$$

Intuitive Idea

$$p(x) = \int p(x|z)p(z)dz$$

- E-Step:

Calculate $q(z^i|x^i) = \frac{p(z^i)p(x^i|z^i)}{\int p(z^i)p(x^i|z^i)dz^i}$

- M-Step:

$$\max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\Big(\log p(x^i|z^i)p(z^i)\Big)dz^i$$

Intuitive Idea

$$p(x) = \int p(x|z)p(z)dz$$

- E-Step:

Calculate $q(z^i|x^i) = \frac{p(z^i)p(x^i|z^i)}{\int p(z^i)p(x^i|z^i)dz^i}$

- M-Step:

$$\max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\Big(\log p(x^i|z^i)p(z^i)\Big)dz^i$$

Intuitive Idea

$$p(x) = \int p(x|z)p(z)dz$$

- E-Step:

Calculate $$q(z^i|x^i) = \frac{p(z^i)p(x^i|z^i)}{\int p(z^i)p(x^i|z^i)dz^i}$$

- M-Step:

$$\max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\left(\log p(x^i|z^i)p(z^i)\right)dz^i$$

Revisit K-means

- K-means Objective

$$\min_{\{\boldsymbol{\mu}\},\{\mathbf{y}\}} J(\{\boldsymbol{\mu}\},\{\mathbf{y}\}) = \min_{\{\boldsymbol{\mu}\},\{\mathbf{y}\}} \sum_{n=1}^{N} \sum_{k=1}^{K} y_k^{(n)} \|\boldsymbol{\mu}_k - \mathbf{x}^{(n)}\|^2$$

$$\text{s.t. } \sum_{k} y_k^{(n)} = 1, \forall n, \text{where } y_k^{(n)} \in \{0,1\}, \forall k, n$$

Intuitive Idea

$$p(x) = \int p(x|z)p(z)dz$$

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\Big(\log p(x^i|z^i)p(z^i) \Big) dz^i$$

Intuitive Idea

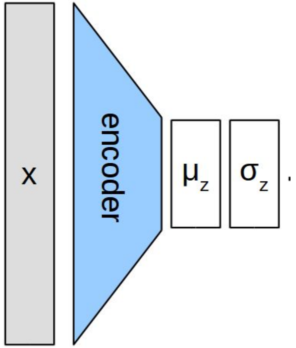
$$p(x) = \int p(x|z)p(z)dz$$

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\Big(\log p(x^i|z^i)p(z^i)\Big)dz^i \underbrace{- \int q(z^i|x^i)\log q(z^i|x^i)dz_i}_{H(q(z|x))}$$

Intuitive Idea

$$p(x) = \int p(x|z)p(z)dz$$

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\left(\log p(x^i|z^i)p(z^i)\right)dz^i - \int q(z^i|x^i)\log q(z^i|x^i)dz_i$$

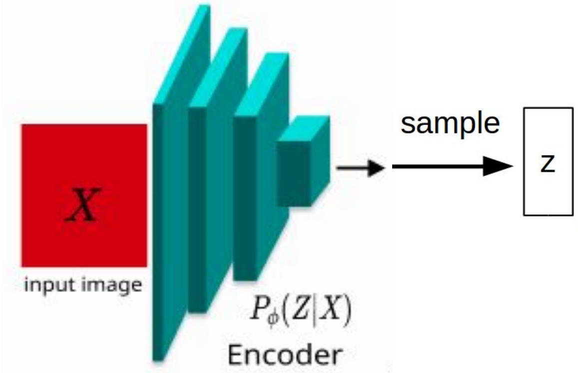


$$q(z|x) = \mathcal{N}\left(z \mid \mu_z(x), \operatorname{diag}(\sigma_z^2(x))\right)$$

Intuitive Idea

$$p(x) = \int p(x|z)p(z)dz$$

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\Big(\log p(x^i|z^i)p(z^i)\Big)dz^i - \int q(z^i|x^i) \log q(z^i|x^i) dz_i$$



$$q(z|x) = \mathcal{N}\left(z \mid \mu_z(x), \mathrm{diag}(\sigma_z^2(x))\right)$$

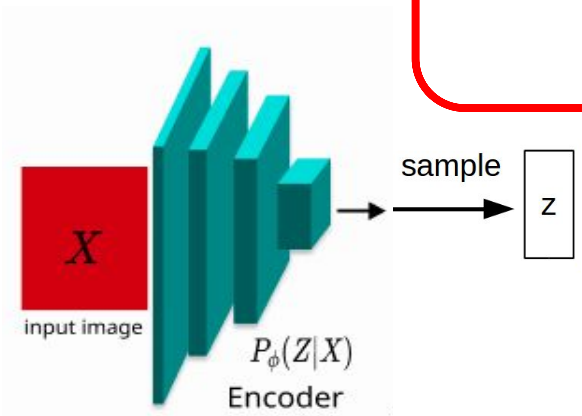
deep neural network

$$\mu_{W_z}(x), \quad \sigma_{W_z}(x)$$

Intuitive Idea

$$p(x) = \int p(x|z)p(z)dz$$

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\left(\log p(x^i|z^i)p(z^i)\right)dz^i - \int q(z^i|x^i) \log q(z^i|x^i) dz_i$$



$$q(z|x) = \mathcal{N}\left(z \mid \mu_z(x), \mathrm{diag}(\sigma_z^2(x))\right)$$

deep neural network

$$\mu_{W_z}(x), \quad \sigma_{W_z}(x)$$

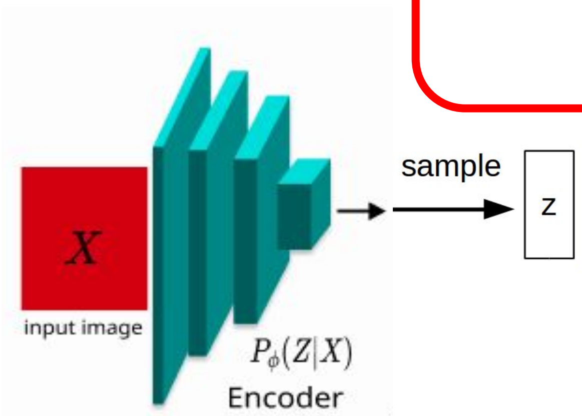
Intuitive Idea

$$p(x) = \int p(x|z)p(z)dz$$

$$\mathbb{E}_{q(z^i|x^i)}\left[\log p(x^i|z^i)p(z^i)\right]$$

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\left(\log p(x^i|z^i)p(z^i)\right)dz^i \quad \mathbb{E}_{q(z^i|x^i)}\left[\log q(z^i|x^i)\right]$$

$$- \int q(z^i|x^i) \log q(z^i|x^i) dz_i$$



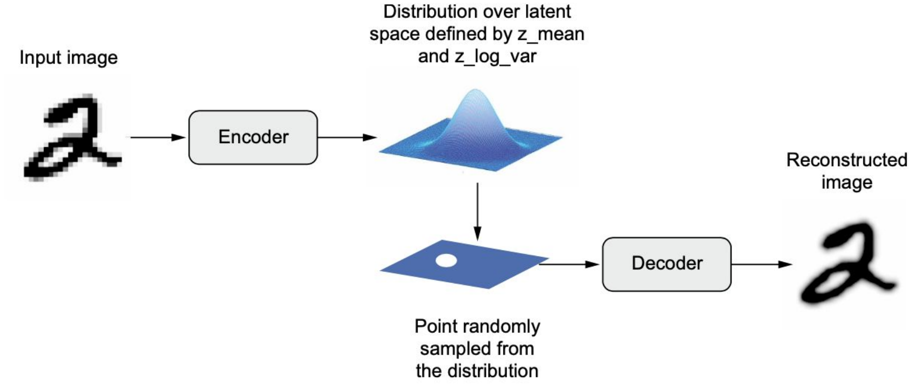
$$q(z|x) = \mathcal{N}\left(z \mid \mu_z(x), \mathrm{diag}(\sigma_z^2(x))\right)$$

deep neural network

$$\mu_{W_z}(x), \quad \sigma_{W_z}(x)$$

Intuitive Idea: Auto-Encoder

$$\max_{W_z^1, W_z^2} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\left(\log p(x^i|z^i)p(z^i)\right)dz^i \underbrace{- \int q(z^i|x^i)\log q(z^i|x^i)dz_i}_{H(q(z|x))}$$



Evidence Lower Bound (of MLE)

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\left(\log p(x^i|z^i)p(z^i) \right) dz^i + H(q(z|x))$$

$$= \max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i)\left(\log \frac{p(x^i|z^i)p(z^i)}{q(z^i|x^i)} \right) dz^i$$

Evidence Lower Bound

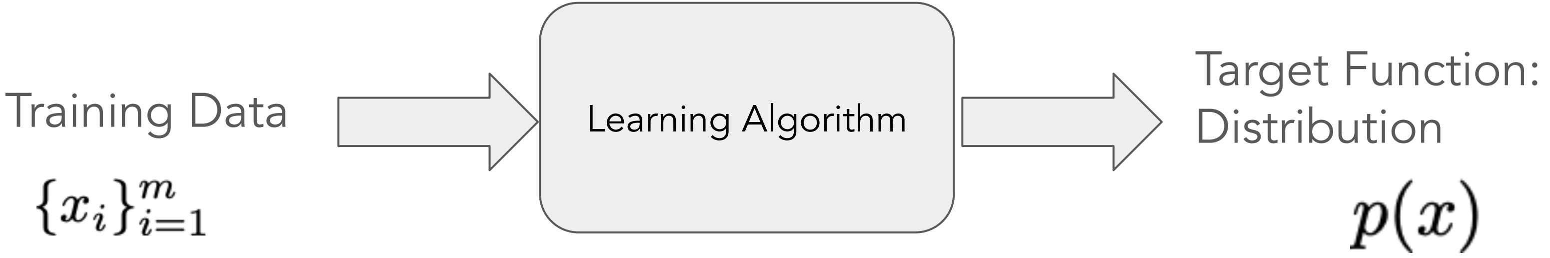
$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i) \Big(\log p(x^i|z^i) p(z^i) \Big) dz^i + H(q(z|x))$$

$$= \max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i) \left(\log \frac{p(x^i|z^i) p(z^i)}{q(z^i|x^i)} \right) dz^i$$

$$\leq \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \log \int \left(\frac{p(x^i|z^i) p(z^i)}{q(z^i|x^i)} q(z^i|x^i) dz^i \right)$$

$$\mathbb{E}[\log Y] \leq \log \mathbb{E}[Y]$$

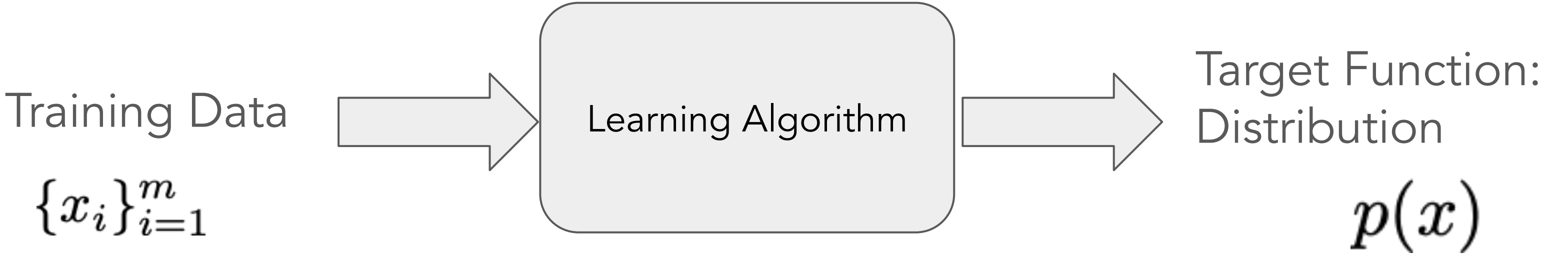
Generative Model: Latent Variable Models



Density Estimation Pipeline

1. Build probabilistic models
2. Derive loss function (by MLE or MAP....)
   MLE -> ELBO
3. Select optimizer

Generative Model: Latent Variable Models



Density Estimation Pipeline

1. Build probabilistic models
2. Derive loss function (by MLE or MAP….)
   MLE -> ELBO
3. Select optimizer
   Stochastic Gradient

Evidence Lower Bound

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i) \Big(\log p(x^i|z^i)p(z^i) \Big) dz^i \\ \underbrace{- \int q(z^i|x^i) \log q(z^i|x^i) dz_i}_{H(q(z|x))}$$

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \mathbb{E}_{q(z^i|x^i)} \Big[\log p(x^i|z^i)p(z^i) - \log q(z^i|x^i) \Big]$$

Reparamerization Trick for Gradient Estimator

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \mathbb{E}_{q(z^i|x^i)} \Big[\log p(x^i|z^i) p(z^i) - \log q(z^i|x^i) \Big]$$

$$z \sim \; q(z|x) = \mathcal{N}\left(z \mid \mu_z(x), \operatorname{diag}(\sigma_z^2(x))\right)$$

Reparamerization Trick

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \mathbb{E}_{q(z^i|x^i)} \left[\log p(x^i|z^i) p(z^i) - \log q(z^i|x^i) \right]$$

$$z \sim \; q(z|x) = \mathcal{N}\left(z \mid \mu_z(x), \mathrm{diag}(\sigma_z^2(x))\right)$$

$$z = \mu_z(x) + \sigma_z(x)\epsilon \qquad\qquad \epsilon \sim \mathcal{N}(0, I)$$

Reparamerization Trick

$$\max_{q(z|x)} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \mathbb{E}_{q(z^i|x^i)} \Big[\log p(x^i|z^i) p(z^i) - \log q(z^i|x^i) \Big]$$

$$z \sim \; q(z|x) = \mathcal{N}\left(z \mid \mu_z(x), \mathrm{diag}(\sigma_z^2(x))\right)$$

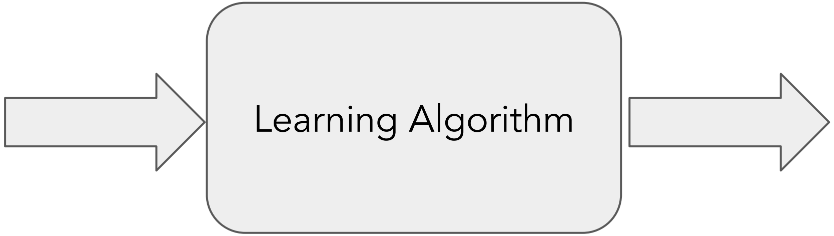
$$z = \mu_z(x) + \sigma_z(x)\epsilon \qquad\qquad \epsilon \sim \mathcal{N}(0, I)$$

$$\max_{W_z^1, W_z^2} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \mathbb{E}_{\epsilon \sim \mathcal{N}(0,I)} \Big[\log p(x^i|\mu(x^i) + \sigma(x^i)\epsilon) p(\mu(x^i) + \sigma(x^i)\epsilon)$$
$$- \log q(\mu(x^i) + \sigma(x^i)\epsilon | x^i) \Big]$$

Generative Model: Latent Variable Models

Training Data

$\{x_i\}_{i=1}^m$



Target Function: Distribution

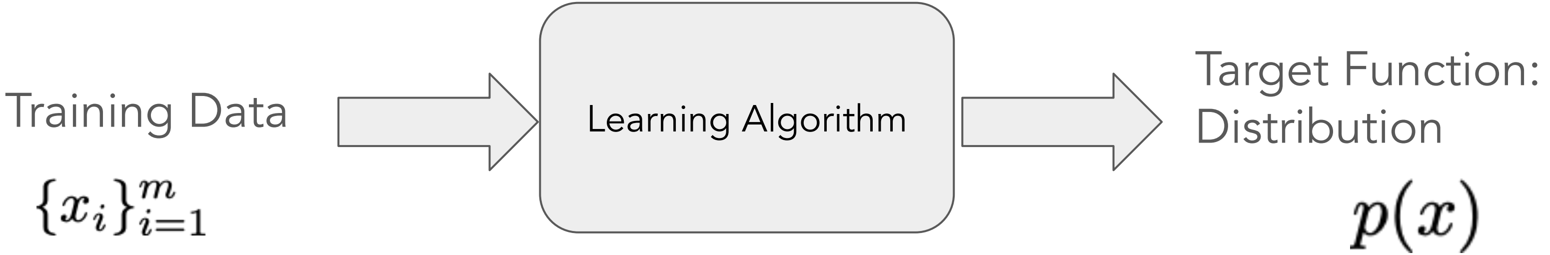
$p(x)$

Density Estimation Pipeline

1. Build probabilistic models
   Deep Latent Variable Model
2. Derive loss function (by MLE or MAP….)
   ELBO
3. Select optimizer
   Stochastic Gradient

VAE Generation

Deep Generative Models: Beyond VAE



Density Estimation Pipeline

1. Build probabilistic models
   Deep Latent Variable Model: Beyond Gaussian
   GAN, Diffusion Models….
2. Derive loss function (by MLE or MAP….)
   ELBO -> Tractable Approximation
3. Select optimizer
   Stochastic Gradient

Q&A

HW3 is out