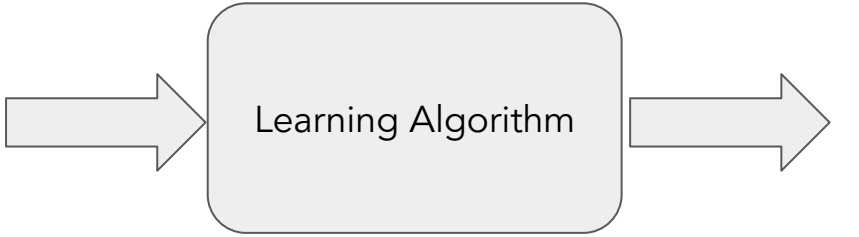
Georgia Tech
College of Computing

CX4240 Spring 2026

Representation Learning

Bo Dai
School of CSE, Georgia Tech
bodai@cc.gatech.edu

Supervised Learning vs. Unsupervised Learning



Training Data

Supervised $\{x^i, y^i\}_{i=1}^m$

Unsupervised $\{x_i\}_{i=1}^m$

Target Function

$f : X \to Y$

Density Estimation $p(x)$

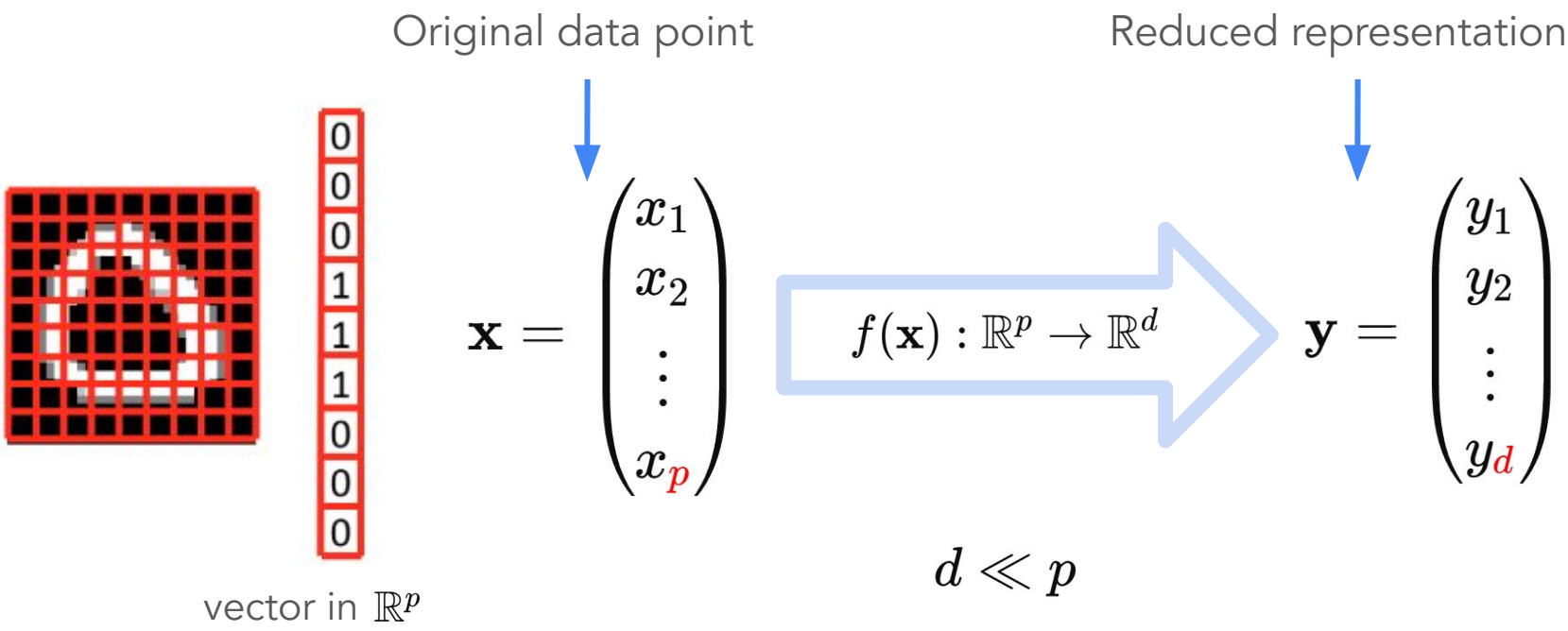
Clustering $f : X \to Y$

$Y \in \{0, 1, \ldots, k\}$

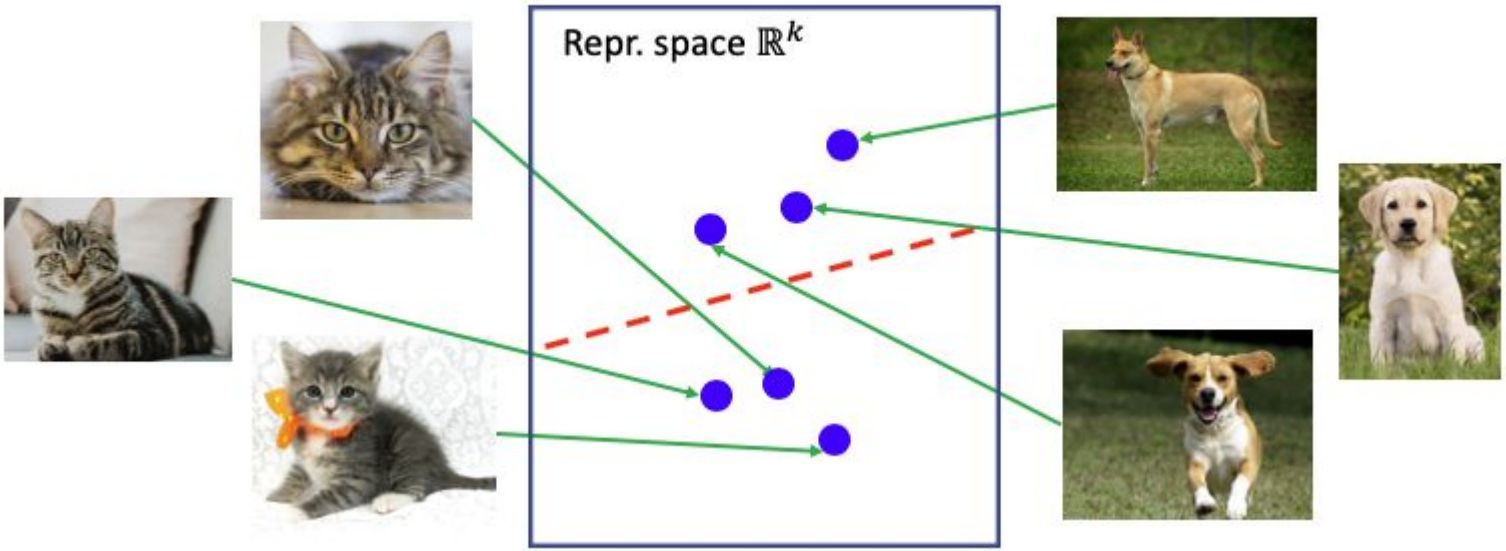
Dimension Reduction $f : X \to Y$

$X \in \mathbb{R}^p, \quad Y \in \mathbb{R}^d$

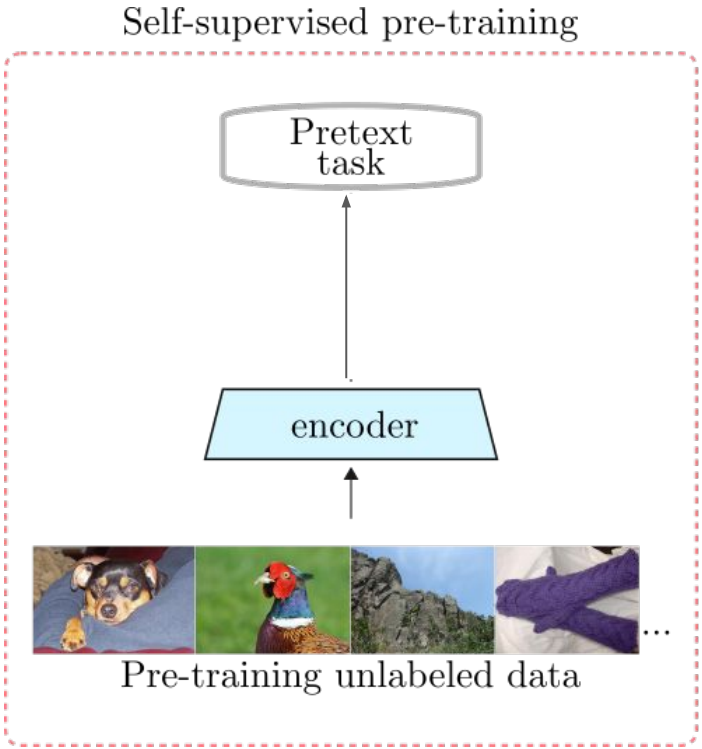
Dimension Reduction/Representation Learning



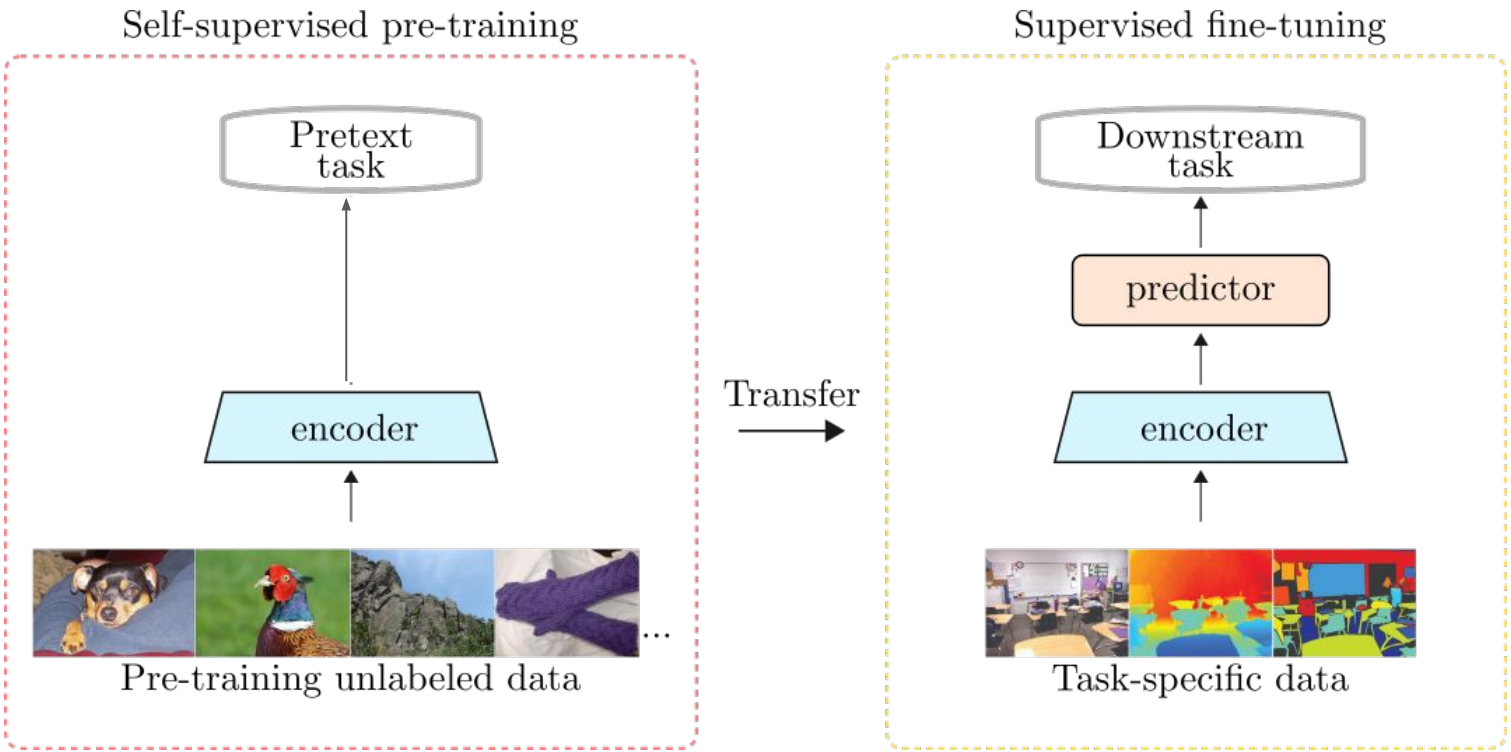
Dimension Reduction/Representation Learning



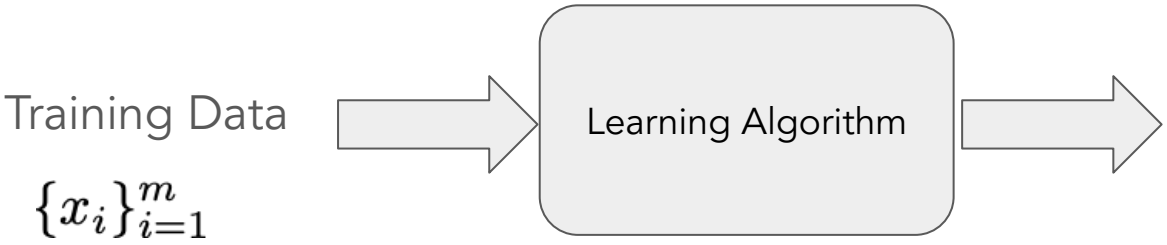
Usage of Representation in ML Tasks



Usage of Representation in ML Tasks



Reconstruction as Pretext Task:
Probabilistic Principal Component Analysis as LVM

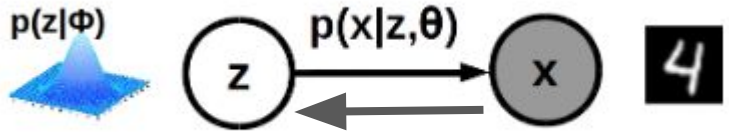


Density Estimation Pipeline

1. Build probabilistic models
   Gaussian Latent Variable Model
2. Derive loss function (by MLE or MAP….)
   MLE
3. Select optimizer
   Necessary Condition

Gaussian LVM for Dimension Reduction

Generation as Pretext Tasks



$$q(z|x)$$

$$p(z) = \mathcal{N}(0, \sigma I)$$

$$p(x|z) = \mathcal{N}(Wz + \mu, \sigma^2 I)$$

$$q(z|x) = \frac{p(x|z)p(z)}{p(x)} = \mathcal{N}(MW^\top(x-\mu), \sigma^2 M)$$

$$M = (W^\top W + \sigma^2 I)^{-1}$$

$$p(x) = \int p(x|z)p(z)dz$$

$$p(x) = \mathcal{N}(\mu, WW^\top + \sigma^2 I)$$

- The posterior mean is given by (see the tutorial)

$$E[z|x] = (W^T W + \sigma^2 I)^{-1} W^T (x - \mu)$$

- Posterior variance:

$$\mathrm{Cov}[z|x] = \sigma^2 (W^T W + \sigma^2 I)^{-1}$$

Chap 2.3 in Pattern Recognition and Machine Learning

Probabilistic PCA

1, Compute $\mu = \frac{1}{N}\sum_{n=1}^{N} x_n \quad S = \frac{1}{N}\sum_{n=1}^{N}(x_n - \mu)(x_n - \mu)^T$

2, Compute Eigen-decomposition

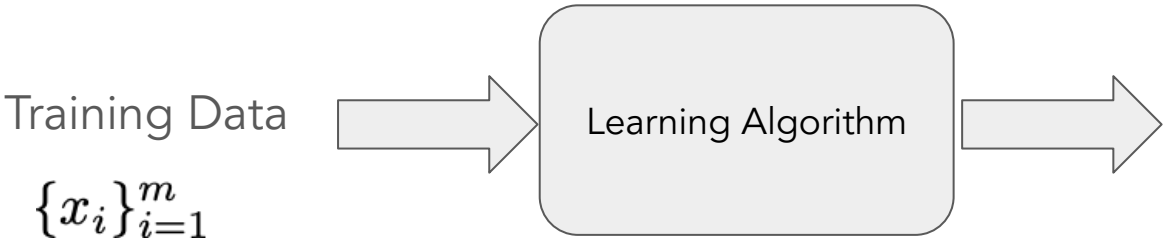
$$S = U_M \Lambda_M U^\top + U_n \Lambda_n U_n^\top$$

$$\sigma^2 = \frac{1}{D-M}\sum_{j=M+1}^{D} \lambda_j$$

3, Compute Projection

$$W = U_M(\Lambda_M - \sigma^2 I)^{1/2}$$

Reconstruction as Pretext Task:
Latent Variable Model for Representation Learning

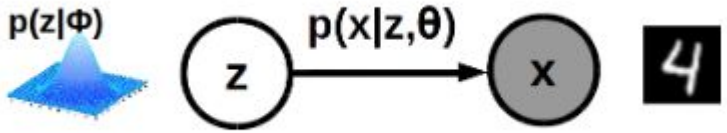


Density Estimation Pipeline

1. Build probabilistic models
   Deep Latent Variable Model
2. Derive loss function (by MLE or MAP….)
   ELBO
3. Select optimizer
   Stochastic Gradient Descent

Revisit Latent Variable Models

Generation as Pretext Tasks

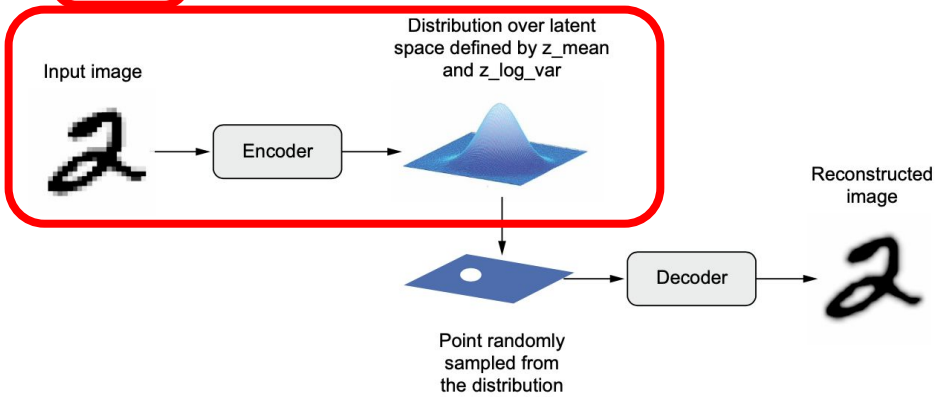


$$p(x) = \int p(x|z)p(z)dz$$

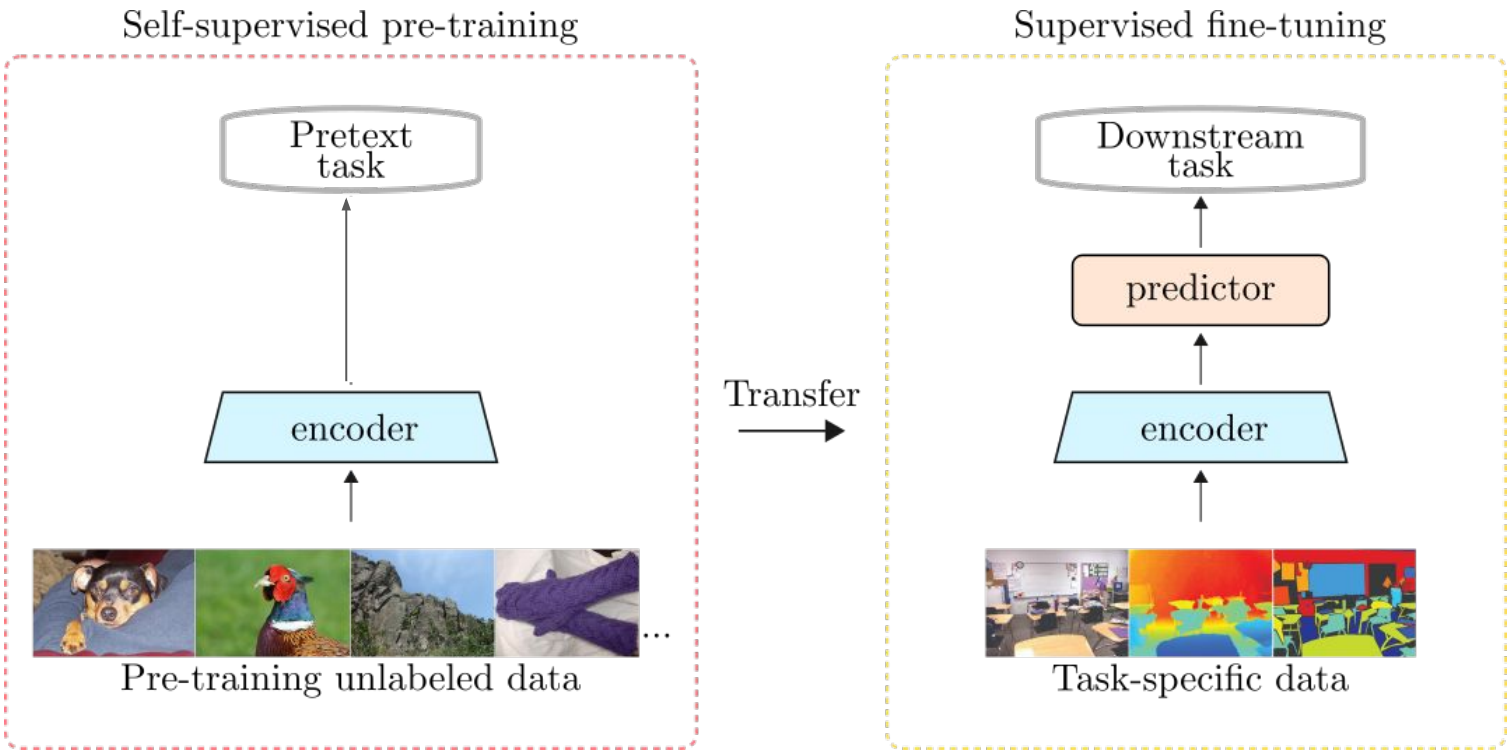
Figure 5: 1024 × 1024 images generated using the CELEBA-HQ dataset. See Appendix F for a larger set of results, and the accompanying video for latent space interpolations.

Generalized LVM for Representation Learning

$$\max_{W_z^1, W_z^2} \max_{\sigma, W_\sigma, W_\mu} \sum_{i=1}^{m} \int q(z^i|x^i) \Big(\log p(x^i|z^i) p(z^i) \Big) dz^i \underbrace{- \int q(z^i|x^i) \log q(z^i|x^i) dz_i}_{H(q(z|x))}$$



Another Pretext Tasks?



Contrastive Representation Learning

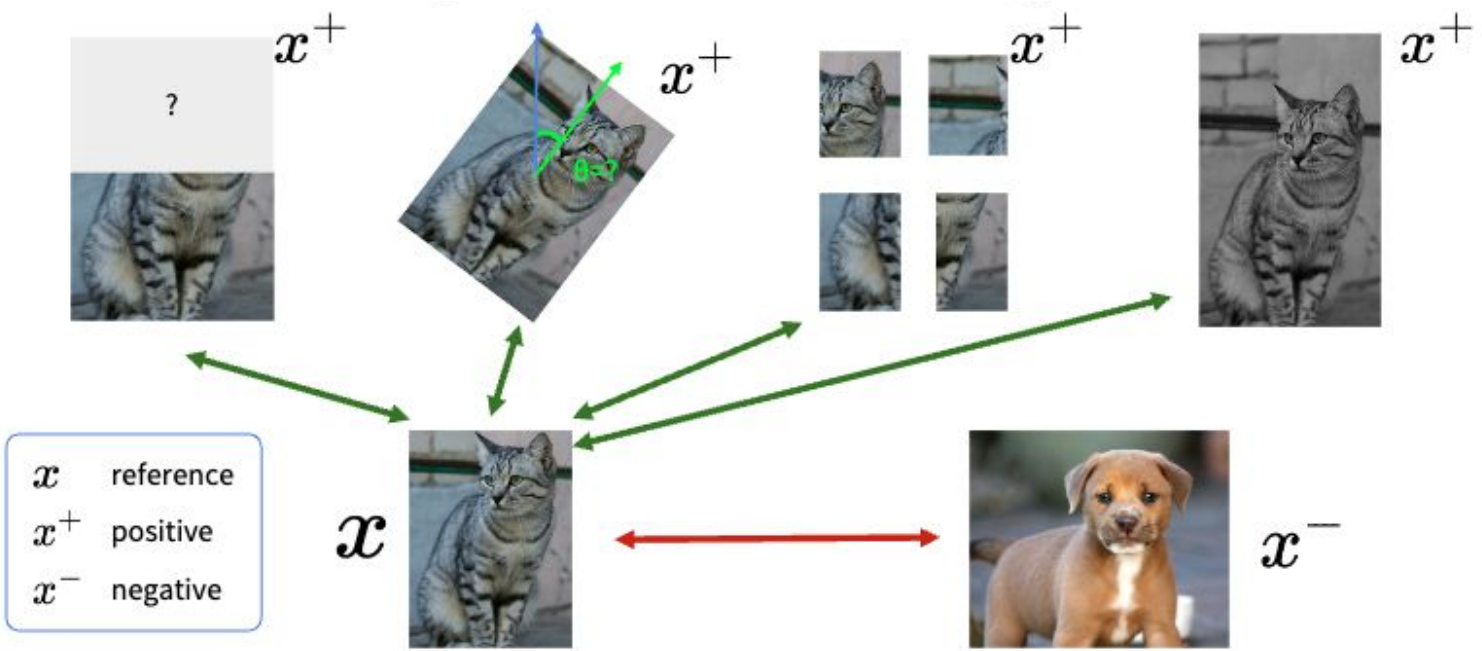
- Basic Idea - Design Pretext Tasks
  - Convert the unsupervised learning to supervised learning

Contrastive Representation Learning

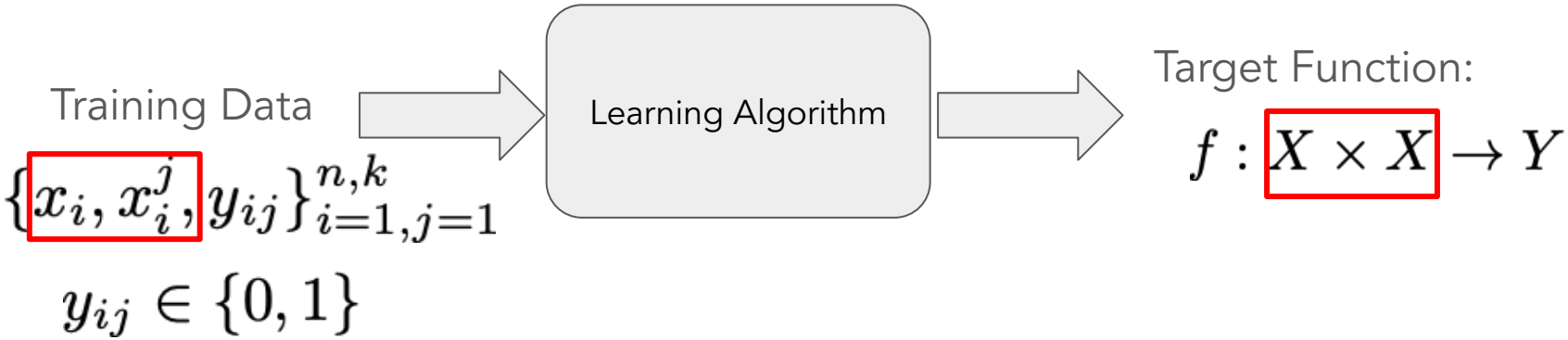
- Basic Idea - Design Pretext Tasks
  - Convert the unsupervised learning to supervised learning

1. Synthesis labels
2. Apply the supervised methods to the synthesis labels
3. Extract the representation

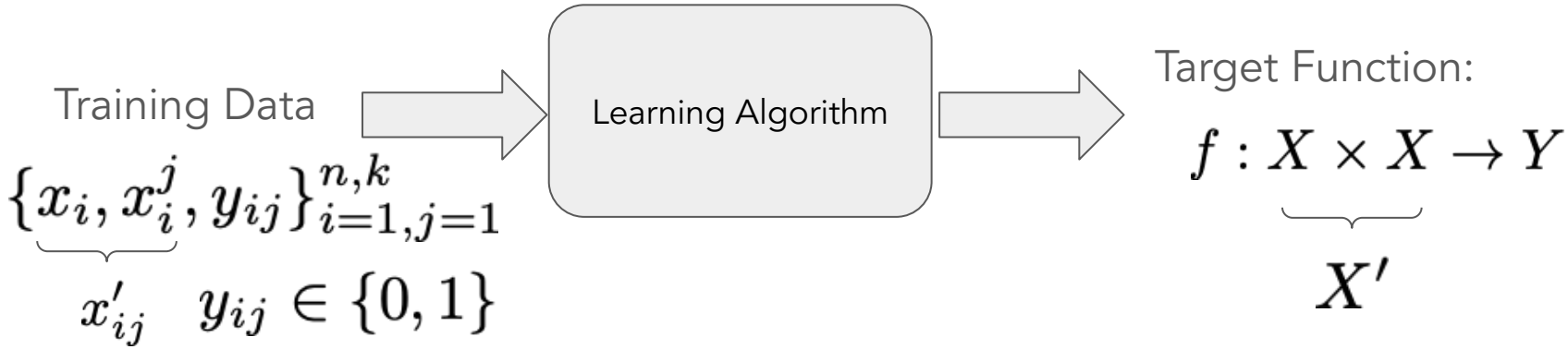
Synthesis Labels



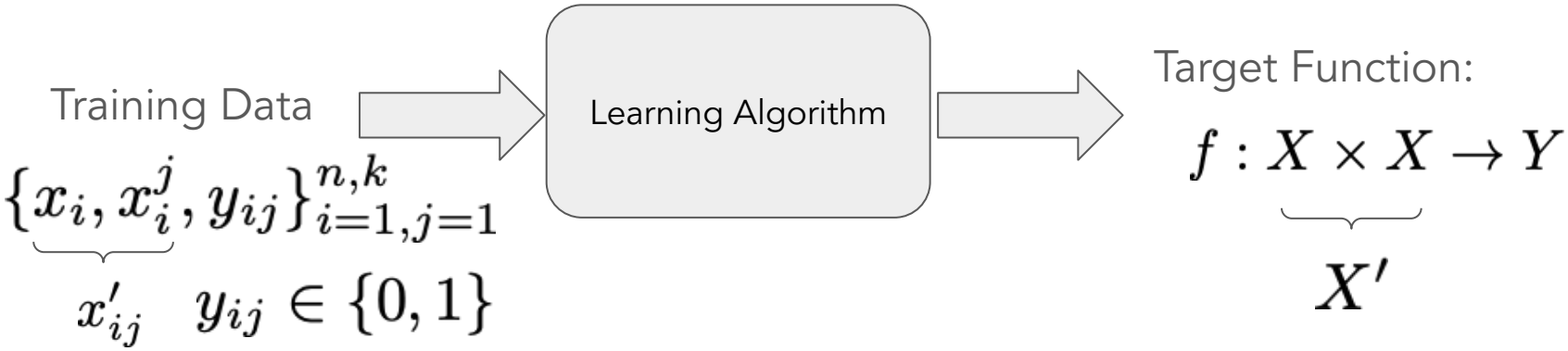
Supervised Tasks



Supervised Tasks: Binary Classification



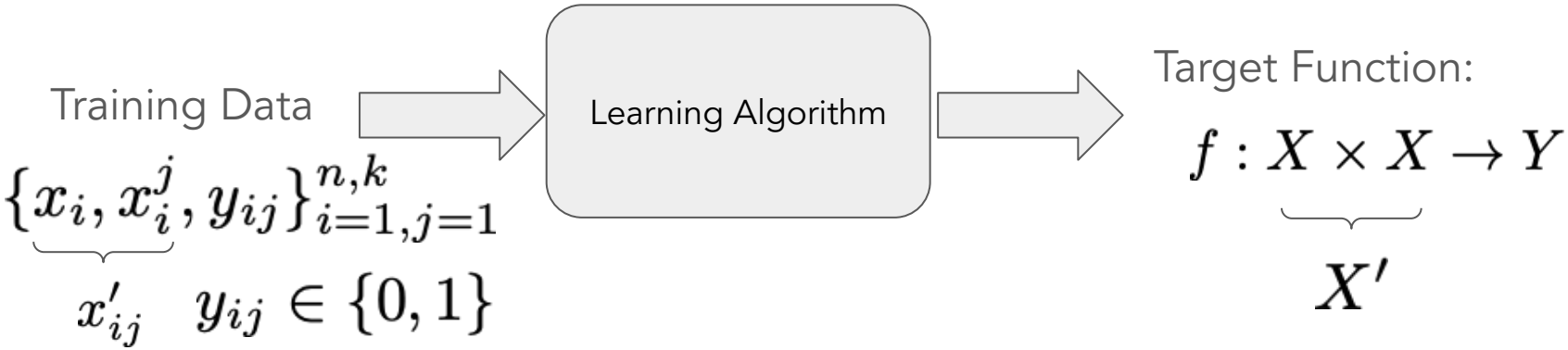
Supervised Tasks: Binary Classification



ML Pipeline

1. Build probabilistic models
2. Derive loss function
3. Select optimizer

Supervised Tasks: Binary Classification



Logistic Regression Pipeline

1. Build probabilistic models: Bernoulli Distribution
2. Derive loss function: MLE and MAP
3. Select optimizer: (Stochastic) Gradient Descent

Probabilistic Model in Classification: Bernoulli Likelihood

$$p(y) = p^y(1-p)^{(1-y)} \qquad p \in [0,1]$$

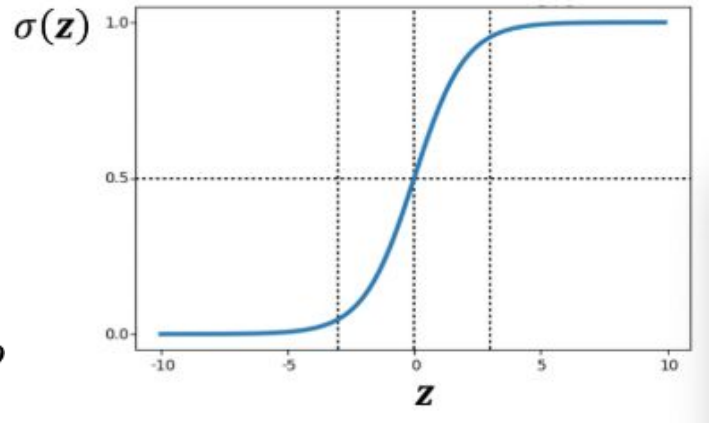
$$p(y|x') = p(y=1|x')^y\{1-p(y=1|x')\}^{1-y}$$

Probabilistic Model in Classification: Bernoulli Likelihood

$$\sigma(z) = \frac{1}{1+e^{-z}} = \frac{e^{z}}{1+e^{z}}$$

$$\phi : X \to S$$

$$X \in \mathbb{R}^d, \quad S \in \mathbb{R}^p$$



$$p(y=1|x') = \sigma(\phi(x_1)^\top \phi(x_2)) = \frac{1}{1+\exp(-\phi(x_1)^\top \phi(x_2))}$$

$$p(y=0|x') = 1 - \frac{1}{1+\exp(-\phi(x_1)^\top \phi(x_2))} = \frac{\exp(-\phi(x_1)^\top \phi(x_2))}{1+\exp(-\phi(x_1)^\top \phi(x_2))}$$

Where is Representation?

Vanilla Binary Logisitic Regression

$$p(y = 1|x, \theta) = \frac{1}{1+\exp(-\theta^\top x)}$$

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Vanilla Binary Logisitic Regression

$$p(y = 1|x, \theta) = \frac{1}{1+\exp(-\theta^\top x)}$$

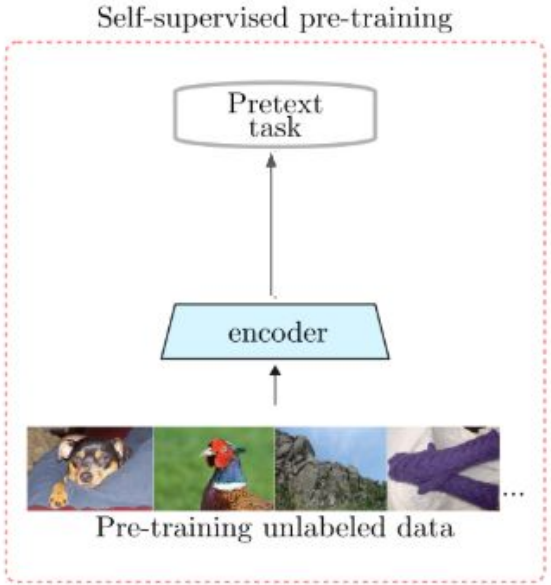
$$p(y = 1|x') = \frac{1}{1 + \exp(-\phi(x_1)^\top \phi(x_2))}$$

Where is Representation?

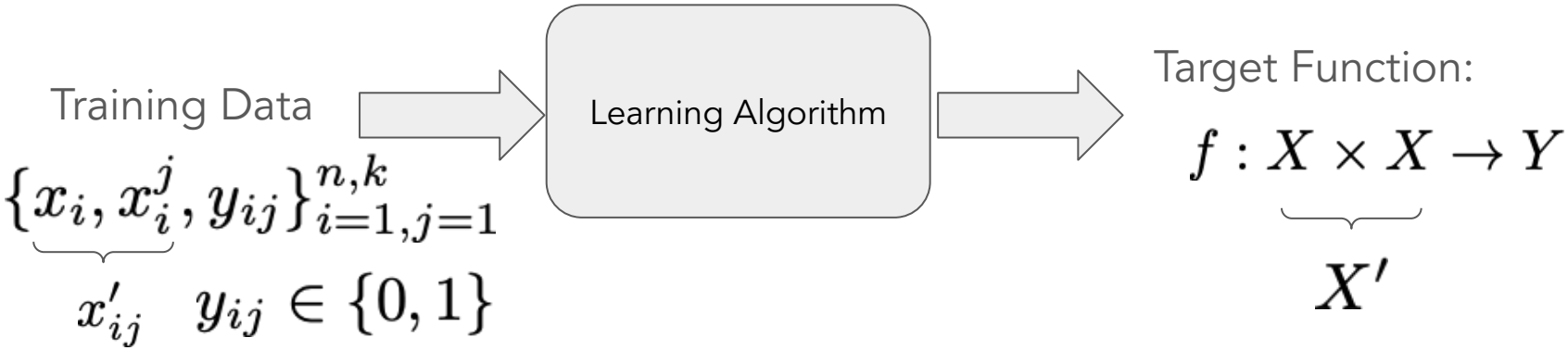
$$p(y = 1|x') = \frac{1}{1 + \exp(-\phi(x_1)^\top \phi(x_2))}$$

$$\phi : X \to S$$

$$X \in \mathbb{R}^d, \quad S \in \mathbb{R}^p$$



Supervised Tasks: Binary Classification



Logistic Regression Pipeline

1. Build probabilistic models: Bernoulli Distribution
2. Derive loss function: MLE and MAP
3. Select optimizer: (Stochastic) Gradient Descent

MLE

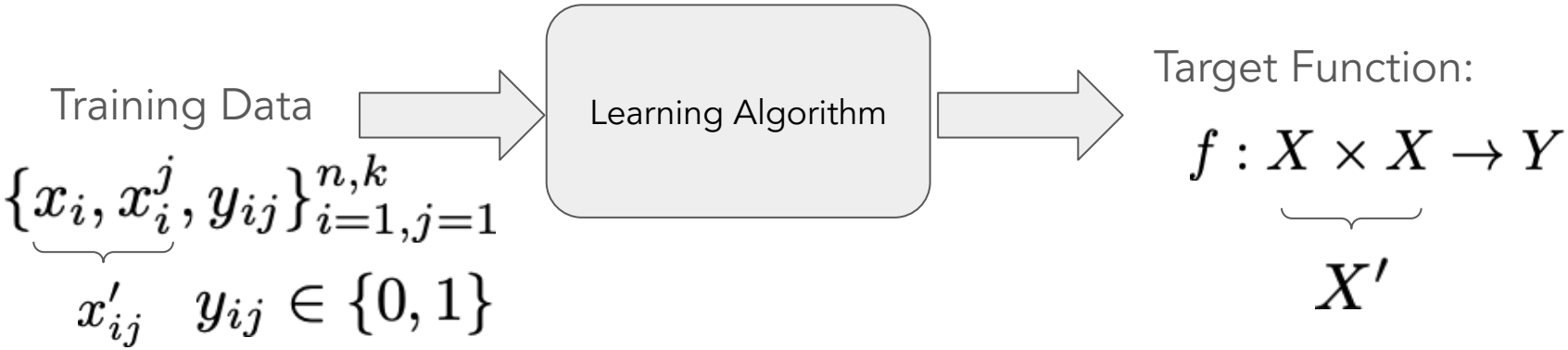
- Logistic regression model

$$p(y=1|x') = \frac{1}{1+\exp(-\phi(x_1)^\top \phi(x_2))}$$

- Plug in

$$\begin{aligned}
\ell(\theta) &:= \log \prod_{i=1}^{n} p(y^i \mid \phi(x_2^i), \phi(x_1^i)) \\
&= \sum_{i=1}^{n} \log \left(\frac{\exp(-\phi(x_1^i)^\top \phi(x_2^i))}{1+\exp(-\phi(x_1^i)^\top \phi(x_2^i))} \right) \cdot \underbrace{I(y^i=0)}_{1-y^i} + \log \left(\frac{1}{1+\exp(-\phi(x_1^i)^\top \phi(x_2^i))} \right) \cdot \underbrace{I(y^i=1)}_{y^i} \\
&= \sum_{i=1}^{n} \left((y^i-1) \cdot \phi(x_1^i)^\top \phi(x_2^i) - \log\left(1+\exp\left(-\phi(x_1^i)^\top \phi(x_2^i)\right)\right) \right)
\end{aligned}$$

Supervised Tasks: Binary Classification



Logistic Regression Pipeline

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Gradient Calculation of MLE

$$\max_{\phi} \log L(\phi) = \sum_{i} (y^i - 1)\, \phi(x_1^i)^\top \phi(x_2^i) - \log\left(1 + \exp\left(-\phi(x_1^i)^\top \phi(x_2^i)\right)\right)$$

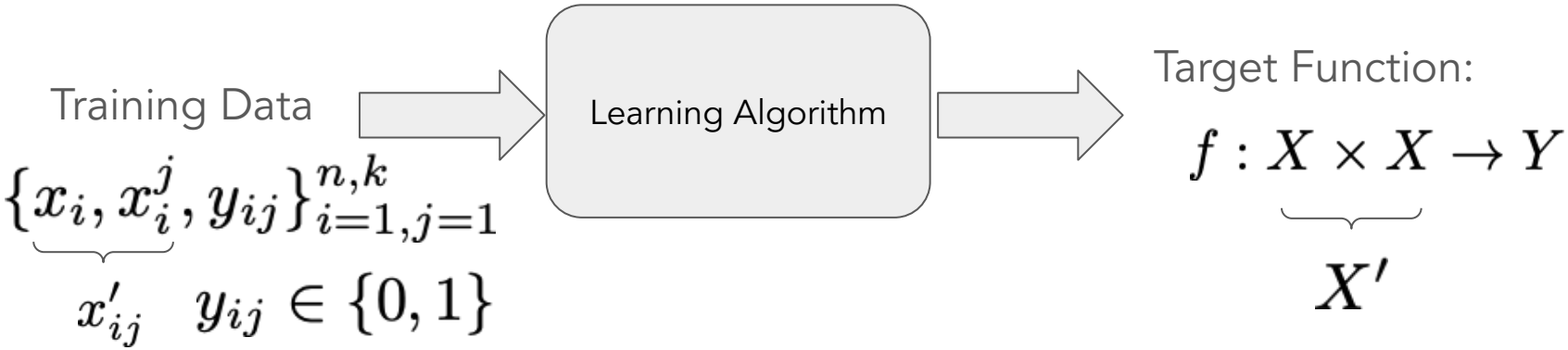
(Stochastic) Gradient Descent

- Initialize parameter ϕ

- Do

$$\phi^{t+1} \leftarrow \phi^t + \eta \nabla \ell(\phi)$$

Supervised Task: Binary Classification



Logistic Regression Pipeline

1. Build probabilistic models: Bernoulli Distribution
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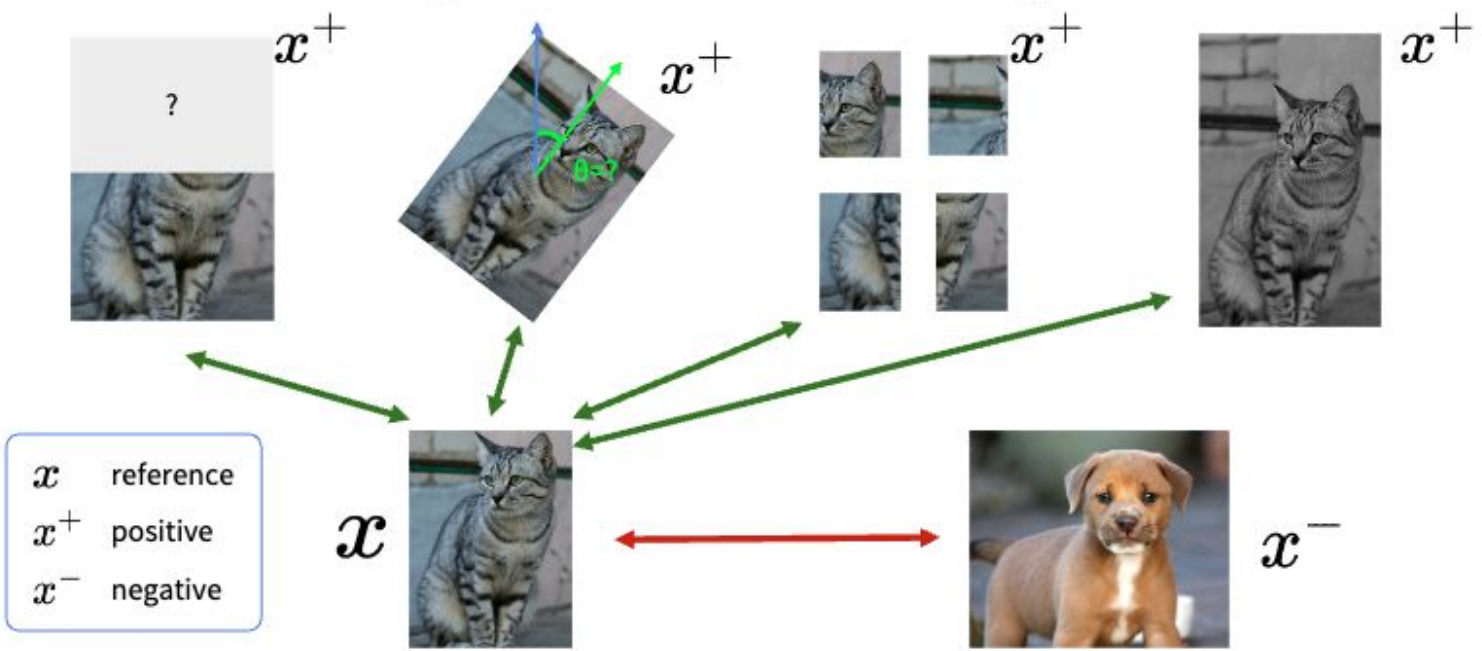
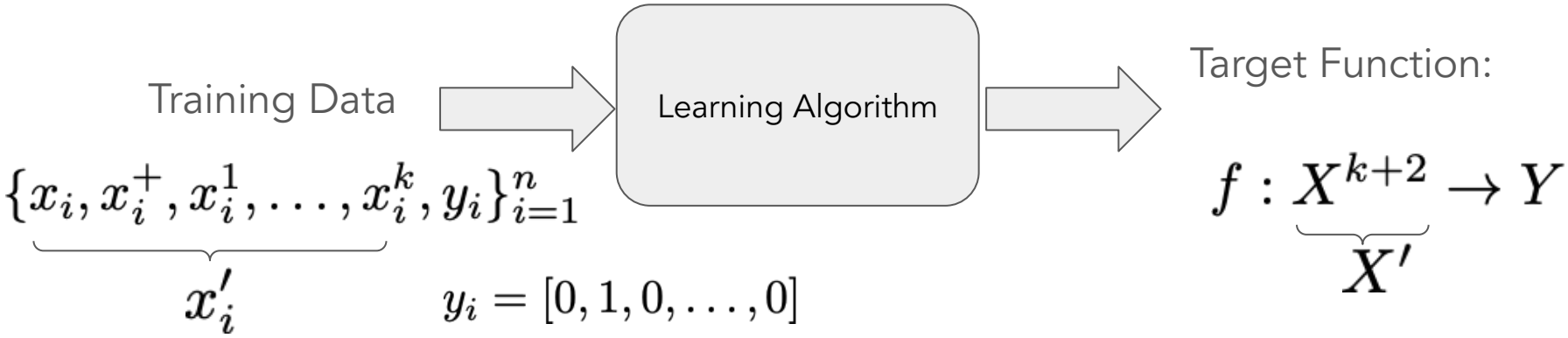
Synthesis Labels



Supervised Tasks: Multiclass Classification



Multiclass Logistic Regression Pipeline

1. Build probabilistic models: Categorical Distribution
2. Derive loss function: MLE and MAP
3. Select optimizer: (Stochastic) Gradient Descent

Probabilistic Model in Multiclass Classification:
Categorical Likelihood

$$p(y = i) = p_i, \quad \sum_{i=1}^{k} p_i = 1, \quad p_i \geq 0$$

$$p(y) = \prod_{i=1}^{k} p_i^{y_i}$$

$$p = (p_1, p_2, \ldots, p_k)$$

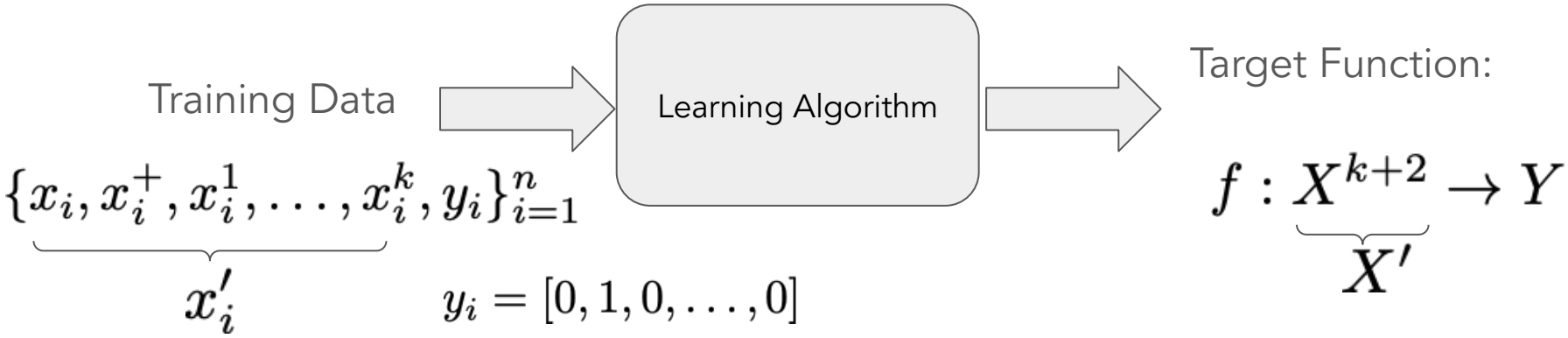
$$y = (y_1, y_2, \ldots, y_k), \quad y_i \in 0, 1, \quad \sum_{i=1}^{k} y_i = 1$$

1-of-k code

Softmax Parametrization

$$p(y^2 = 1|x') = \frac{\exp(\phi(x)^\top \phi(x^+))}{\exp(\phi(x)^\top \phi(x^+)) + \sum_{j=1}^{k} \exp(\phi(x)^\top \phi(x^j))}$$

Supervised Tasks: Multiclass Classification



Multiclass Logistic Regression Pipeline

1. Build probabilistic models: Categorical Distribution
2. Derive loss function: MLE and MAP
3. Select optimizer: (Stochastic) Gradient Descent

MLE

$$\max_{\phi} \log L(\phi) = \log \prod_{i=1}^{n} \prod_{j=0}^{k} p(y_i^j | x_i')$$

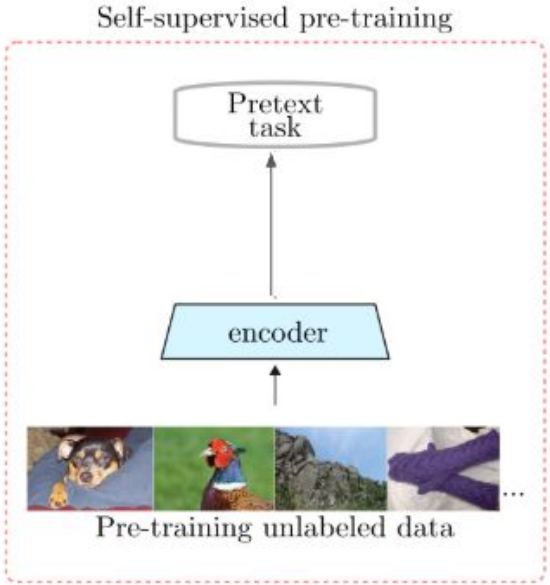
$$= \sum_{i=1}^{n} \log \frac{\exp(\phi(x_i)^\top \phi(x_i^+))}{\exp(\phi(x_i)^\top \phi(x_i^+)) + \sum_{j=1}^{k} \exp(\phi(x_i)^\top \phi(x_i^j))}$$

Where is Representation?

$$p(y^2 = 1|x') = \frac{\exp(\phi(x)^\top \phi(x^+))}{\exp(\phi(x)^\top \phi(x^+)) + \sum_{j=1}^{k} \exp(\phi(x)^\top \phi(x^j))}$$

$$\phi : X \to S$$

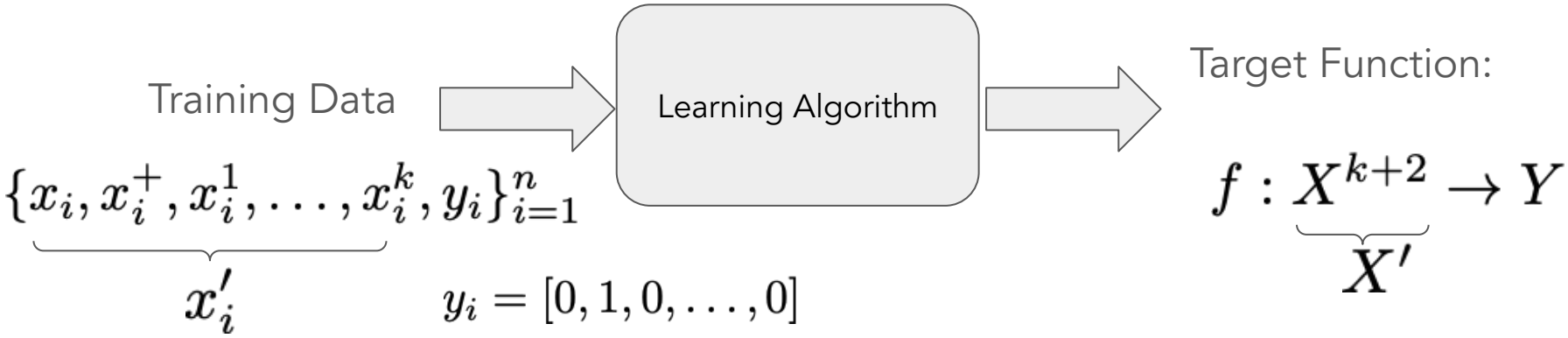
$$X \in \mathbb{R}^d, \quad S \in \mathbb{R}^p$$



Gradient of MLE

$$\sum_{i=1}^{n} \log \frac{\exp(\phi(x_i)^\top \phi(x_i^+))}{\exp(\phi(x_i)^\top \phi(x_i^+)) + \sum_{j=1}^{k} \exp(\phi(x_i)^\top \phi(x_i^j))}$$

SimCLR (ICML 2020): Multiclass Classification



Multiclass Logistic Regression Pipeline

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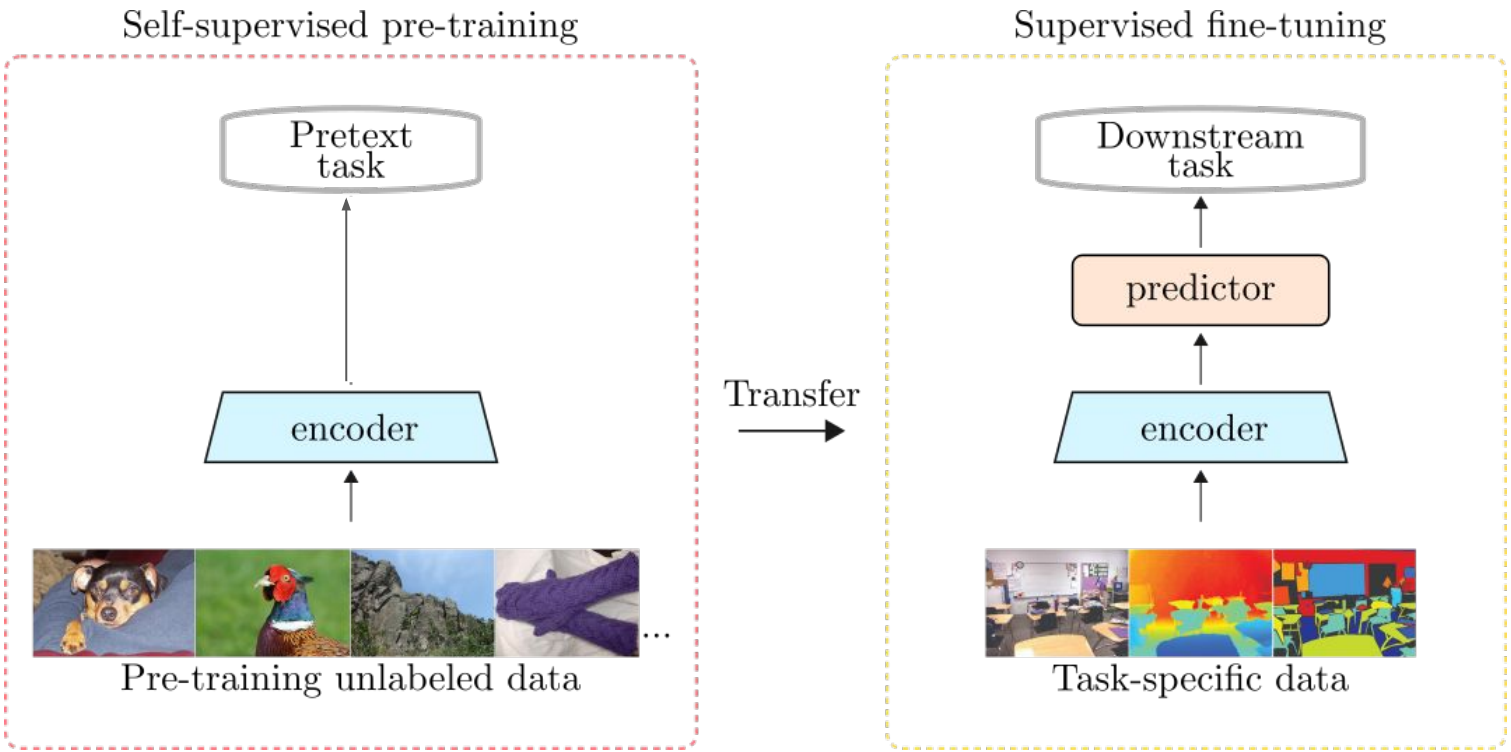
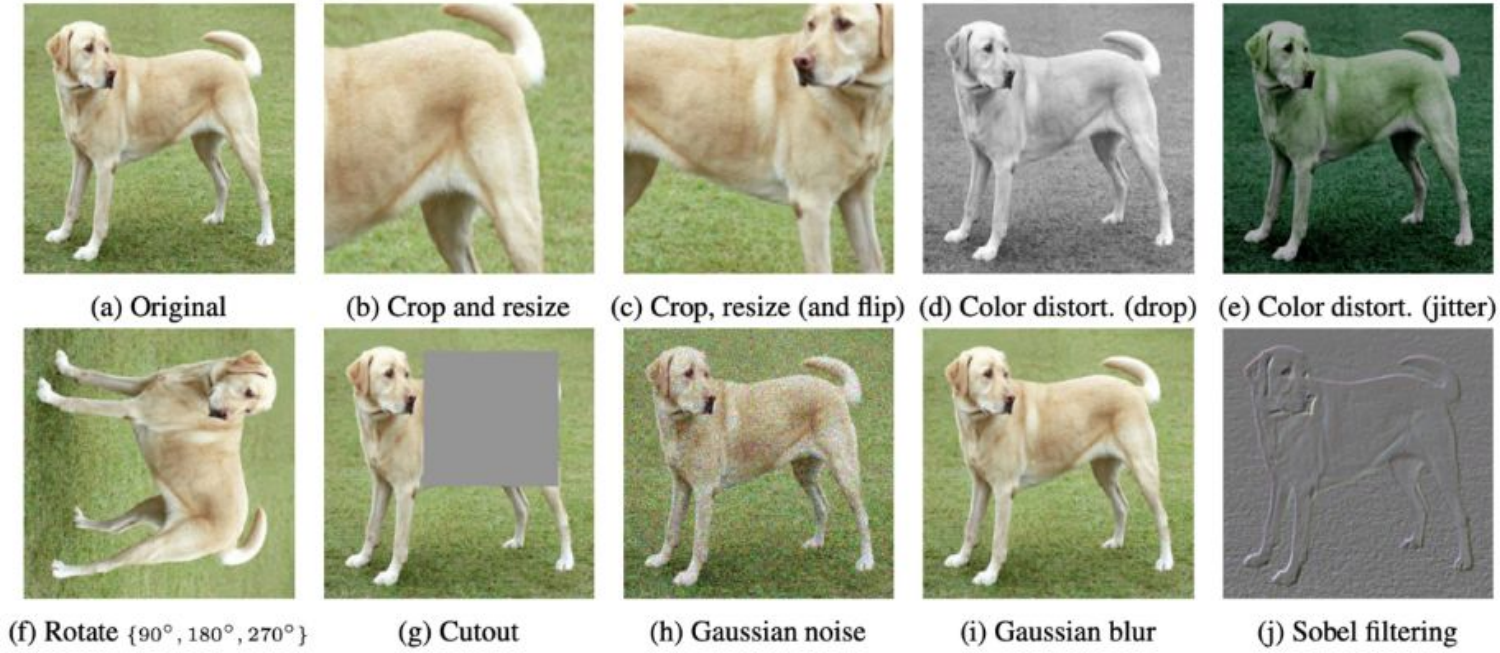
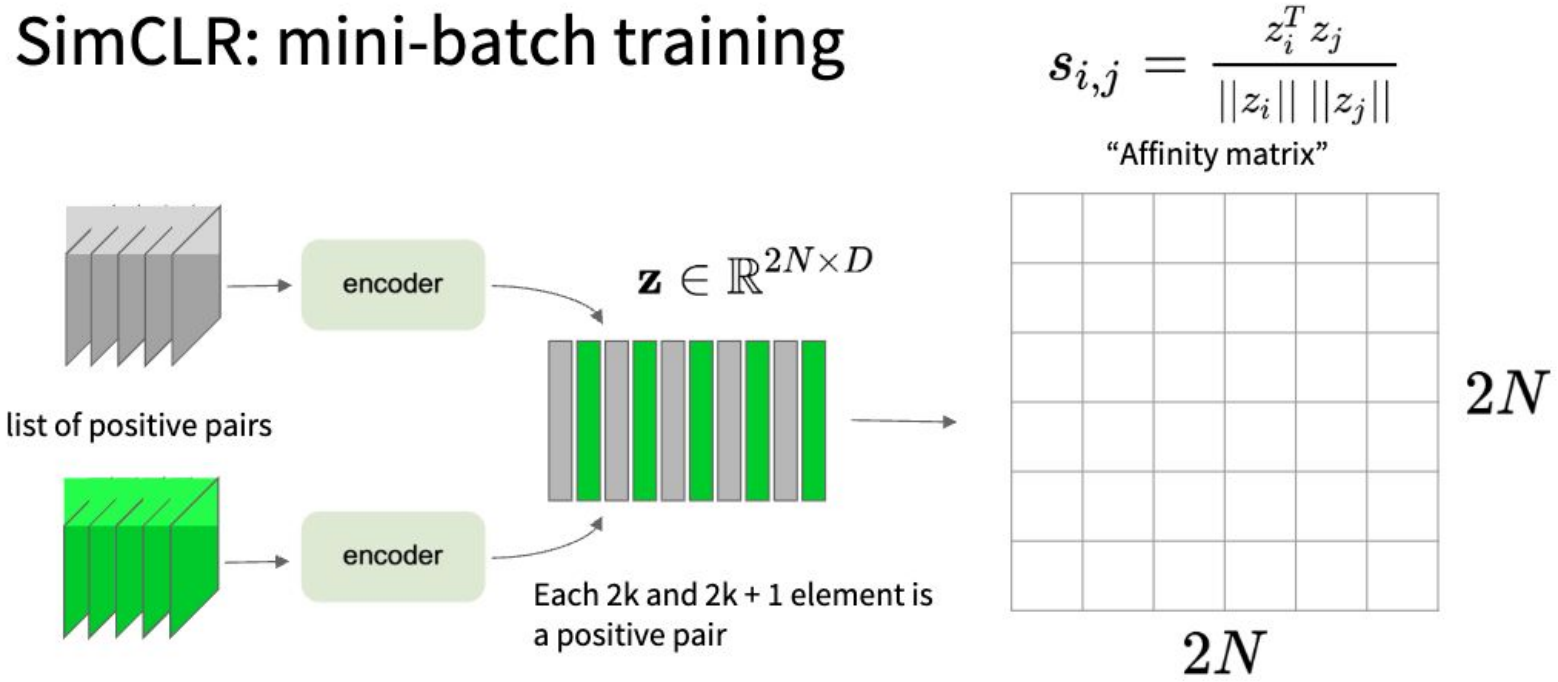
Usage of Representation in ML Tasks



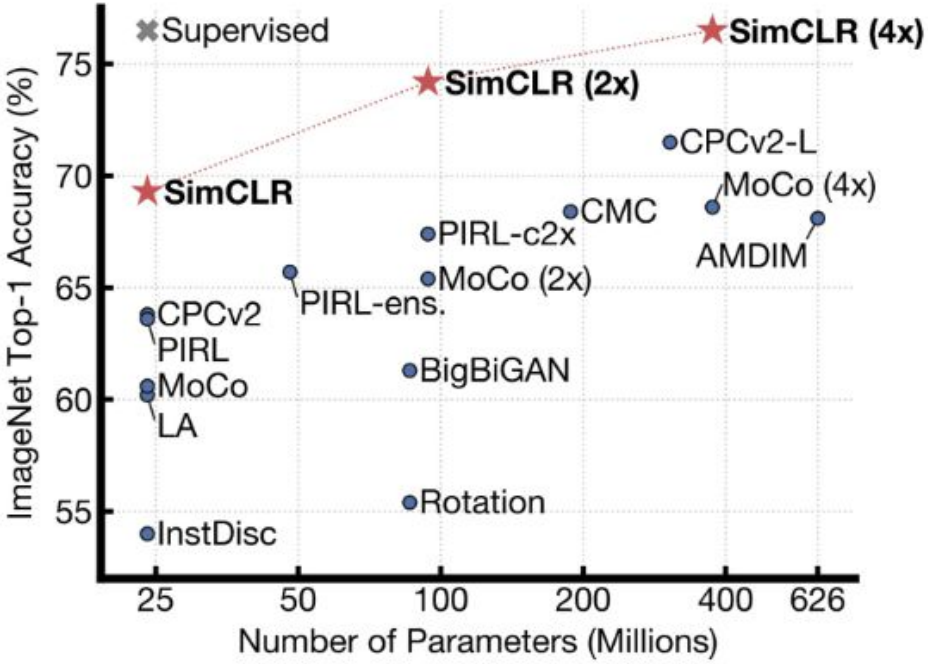
Some Details: Positive Samples



Some Details: Negative Samples



Empirical Performances

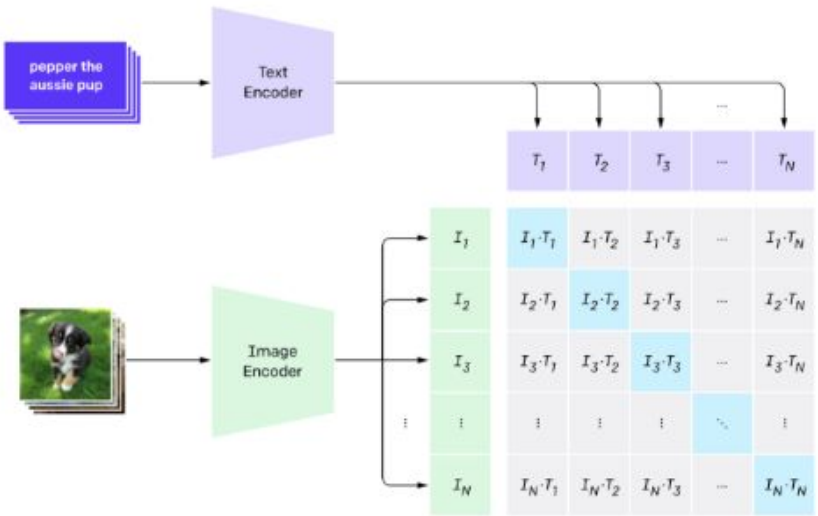


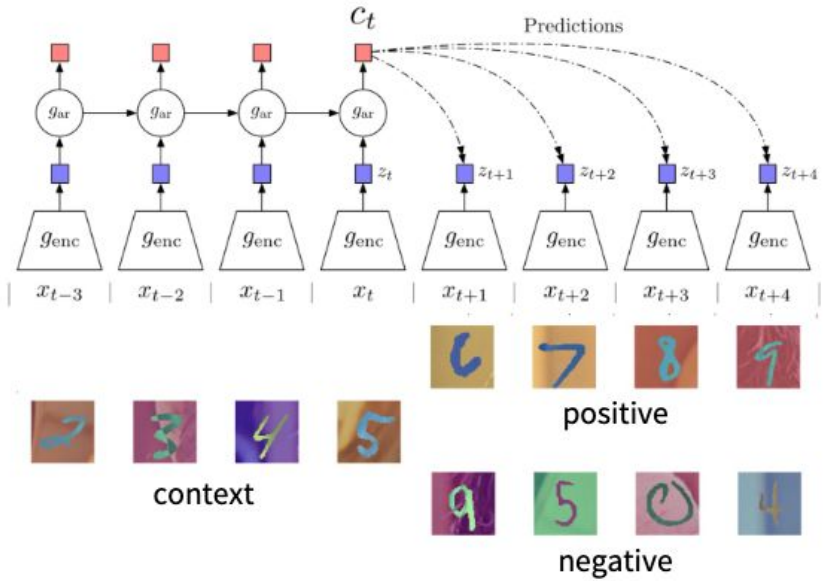
Train feature encoder on ImageNet
(entire training set) using SimCLR.

Freeze feature encoder, train a linear classifier on top with labeled
data.

Extension

- Extension: multi-modality (CLIP), sequences (CPC), Text (word2vec)





Summary

- Representation Learning Pipeline

  Convert the unsupervised learning to supervised learning

  - Synthesis labels
  - Apply the supervised methods to the synthesis labels
  - Extract the representation

- Extension: multi-modality (CLIP), sequences (CPC), Text (word2vec)
- More Variants

Q&A
Please Signup for Project Presentation