Georgia Tech
College of Computing

CX4240 Spring 2026

(Large) Language Model (Part I): Attention and Transformers

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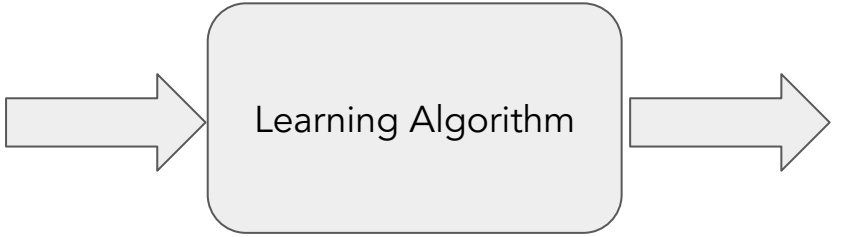
Presentation

- Please signup the presentation date in group
  - Today is the deadline!
  - Group 7, 11, 18, and 21
- Final presentation will be online.
- Make sure to complete in time to get the bonus
  - Think this is a conference talk
- Final report is due on May 4th.

Supervised Learning vs. Unsupervised Learning



Training Data

Supervised $\{x^i, y^i\}_{i=1}^m$

Unsupervised $\{x_i\}_{i=1}^m$

Target Function

$f : X \to Y$

Density Estimation $p(x)$

Clustering $f : X \to Y$

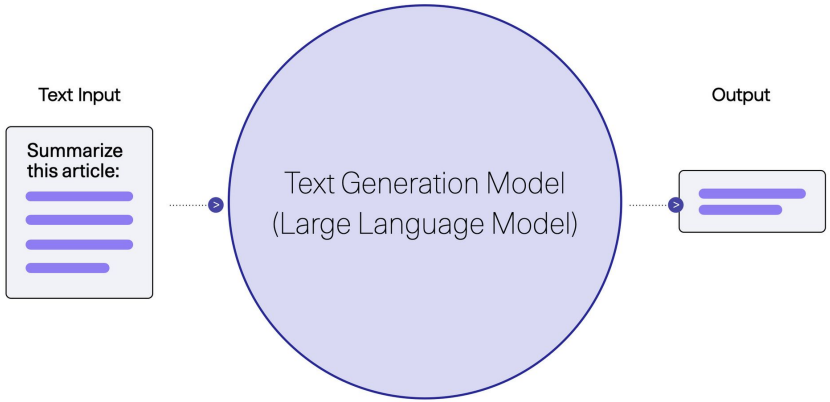
$Y \in \{0, 1, \ldots, k\}$

Dimension Reduction $f : X \to Y$

$X \in \mathbb{R}^p, \quad Y \in \mathbb{R}^d$

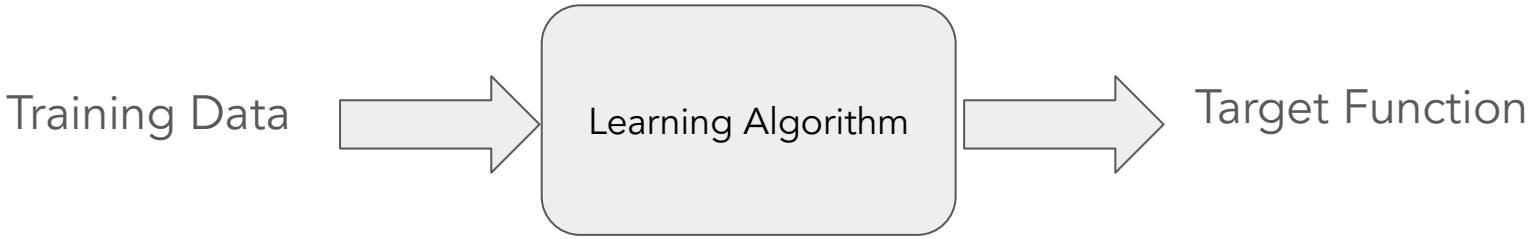
(Large) Language Models

all text data



$$p(x) = \prod_{i=1}^{k} p(x_i | x_{<i})$$

(Large) Language Models



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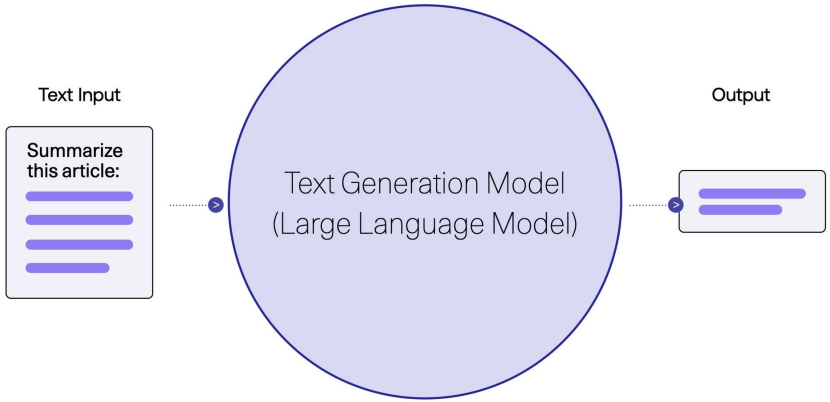
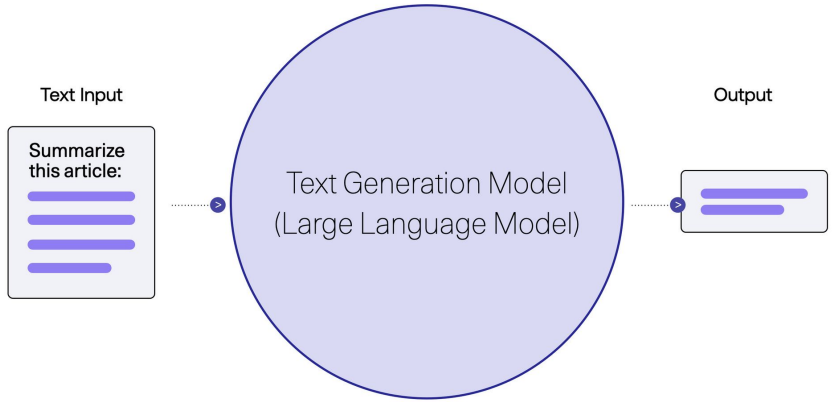


$$p(x_{l,\ldots,k}|x_{<l}) = \prod_{i=l}^{k} p(x_i|x_{<i})$$

(Large) Language Models

Training Data → Learning Algorithm → Target Function

Unlabelled Data: text sequences



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(Large) Language Models

Training Data → Learning Algorithm → Target Function

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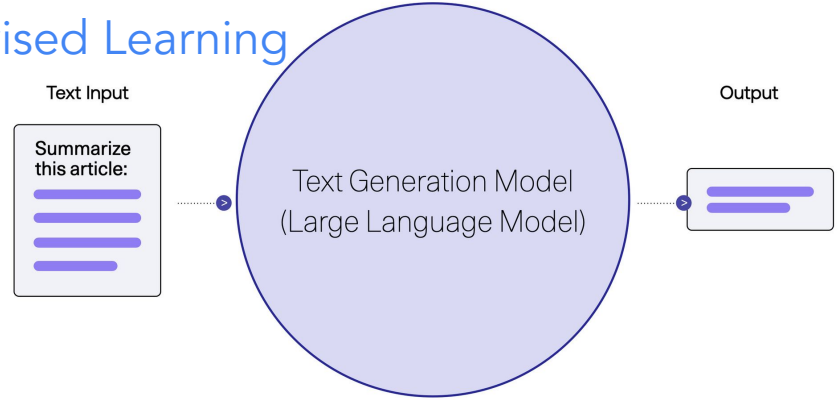
Unsupervised Learning



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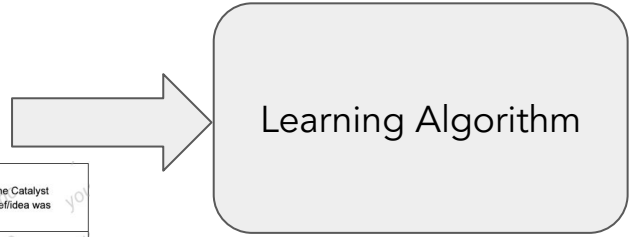
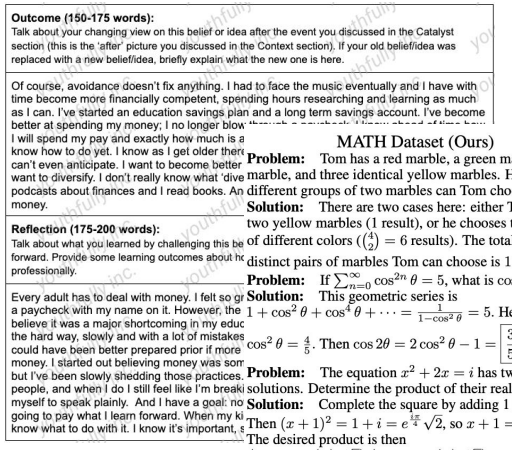
(Large) Language Models

Training Data



Target Function



MATH Dataset (Ours)

Problem: Tom has a red marble, a green marble, a blue marble, and three identical yellow marbles. How many different groups of two marbles can Tom choose?

Solution: There are two cases here: either Tom chooses two yellow marbles (1 result), or he chooses two marbles of different colors ($\binom{4}{2} = 6$ results). The total number of distinct pairs of marbles Tom can choose is $1 + 6 = \boxed{7}$.

Problem: If $\sum_{n=0}^{\infty} \cos^{2n} \theta = 5$, what is $\cos 2\theta$?

Solution: This geometric series is $1 + \cos^2 \theta + \cos^4 \theta + \cdots = \frac{1}{1-\cos^2 \theta} = 5$. Hence, $\cos^2 \theta = \frac{4}{5}$. Then $\cos 2\theta = 2\cos^2 \theta - 1 = \boxed{\frac{3}{5}}$.

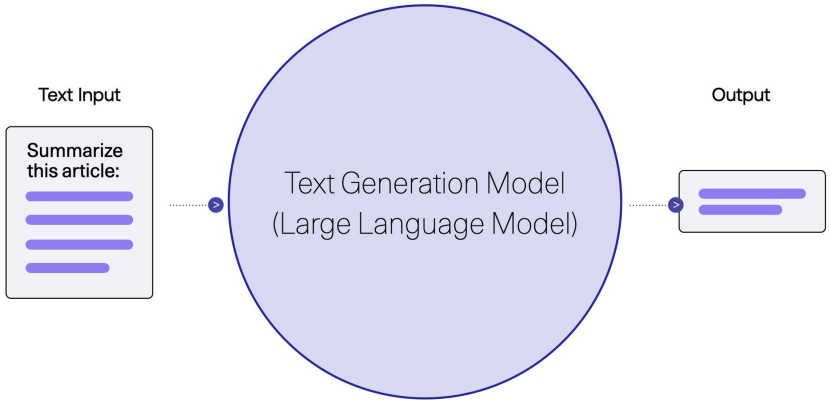
Problem: The equation $x^2 + 2x = i$ has two complex solutions. Determine the product of their real parts.

Solution: Complete the square by adding 1 to each side. Then $(x+1)^2 = 1 + i = e^{\frac{i\pi}{4}}\sqrt{2}$, so $x + 1 = \pm e^{\frac{i\pi}{8}}\sqrt[4]{2}$. The desired product is then

$$\left(-1 + \cos\left(\tfrac{\pi}{8}\right)\sqrt[4]{2}\right)\left(-1 - \cos\left(\tfrac{\pi}{8}\right)\sqrt[4]{2}\right) =$$

$$1 - \cos^2\left(\tfrac{\pi}{8}\right)\sqrt{2} = 1 - \frac{\left(1+\cos\left(\frac{\pi}{4}\right)\right)}{2}\sqrt{2} = \boxed{\frac{1-\sqrt{2}}{2}}.$$

Labelled Data: Prompt & Answer



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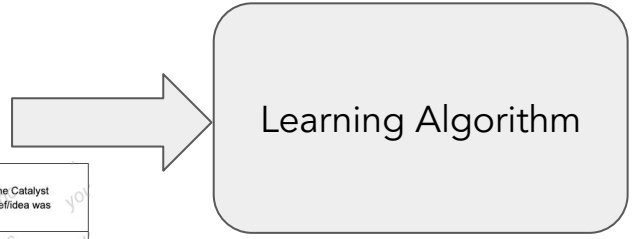
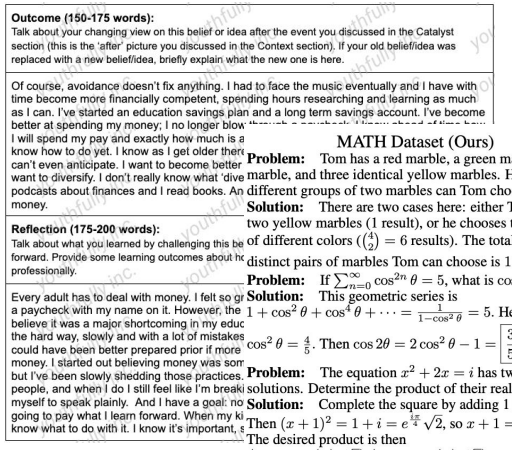
(Large) Language Models



Training Data

Target Function



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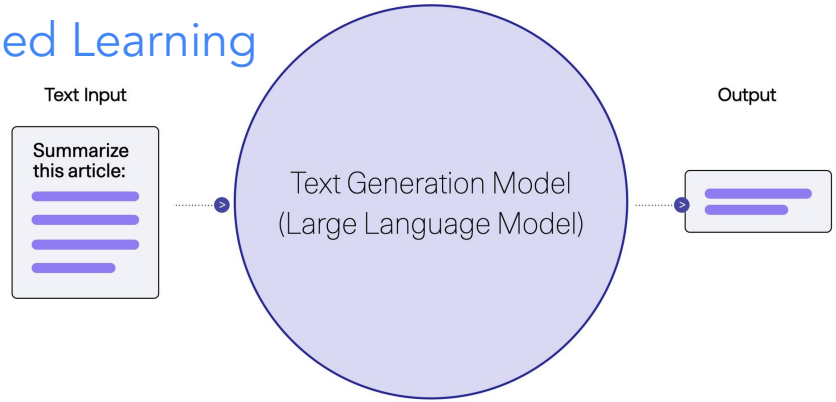
Labelled Data: Prompt & Perfect Answer

Supervised Learning



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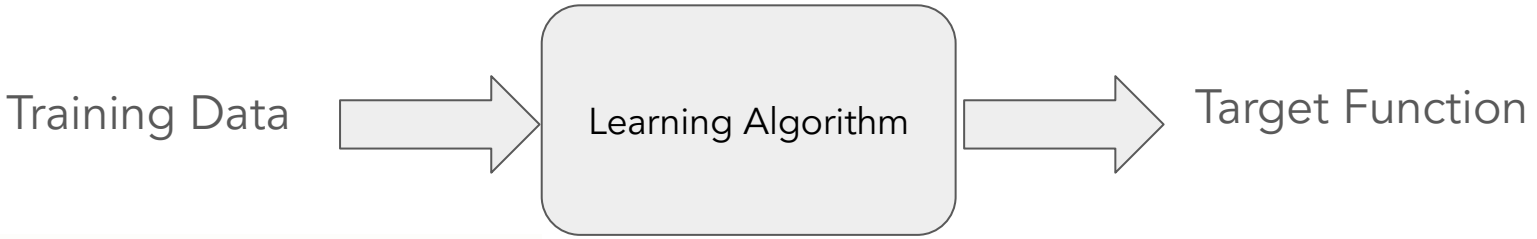
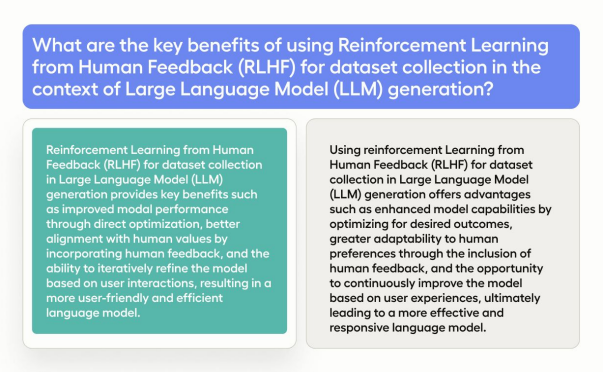
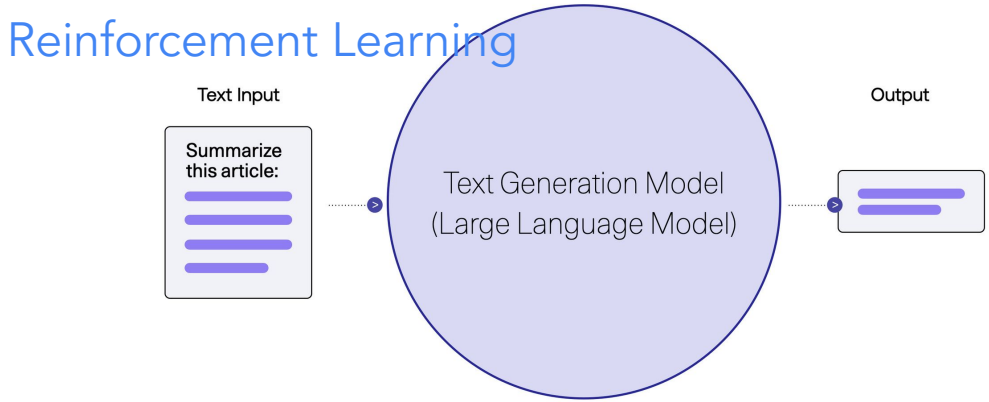
(Large) Language Models



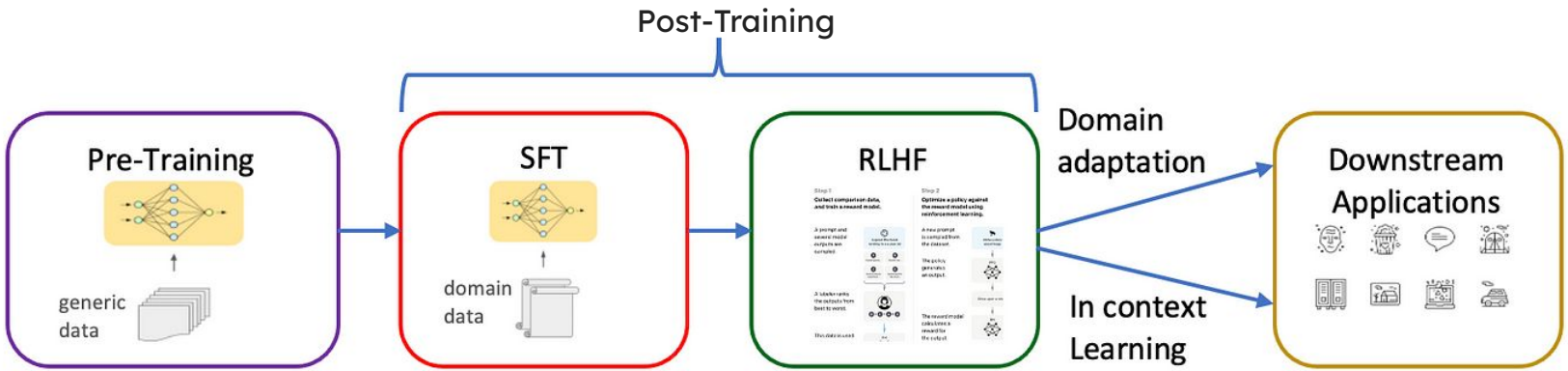




Preference Data:
Prompt & Positive/ Negative Answers

$$p(x_{l,\ldots,k}|x_{<l}) = \prod_{i=l}^{k} p(x_i|x_{<i})$$

LLM Training



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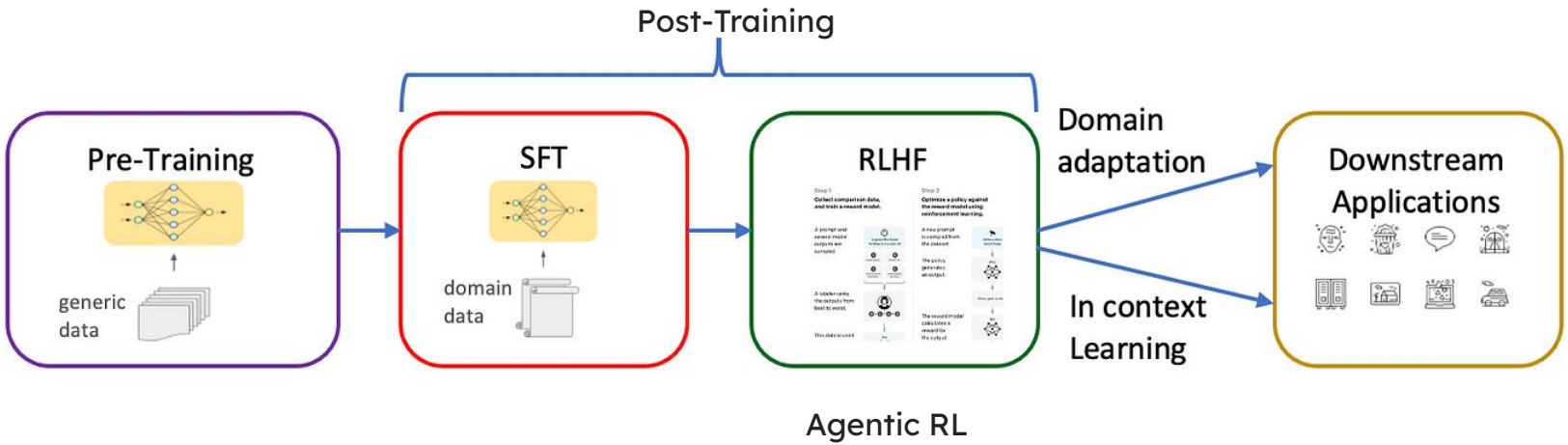

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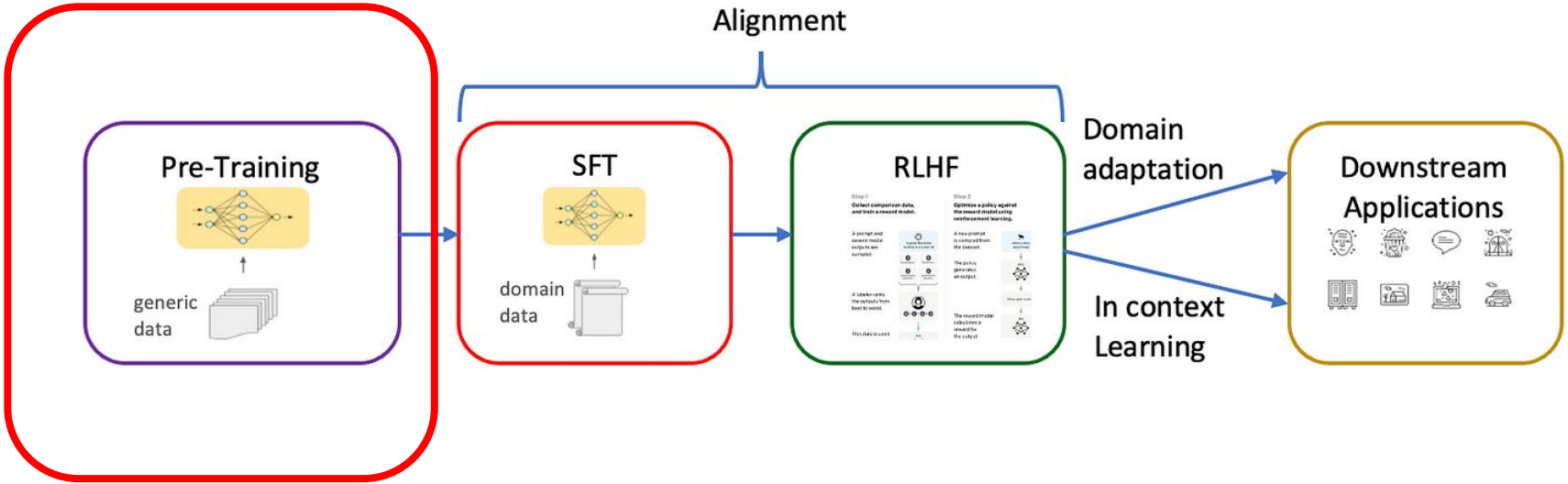
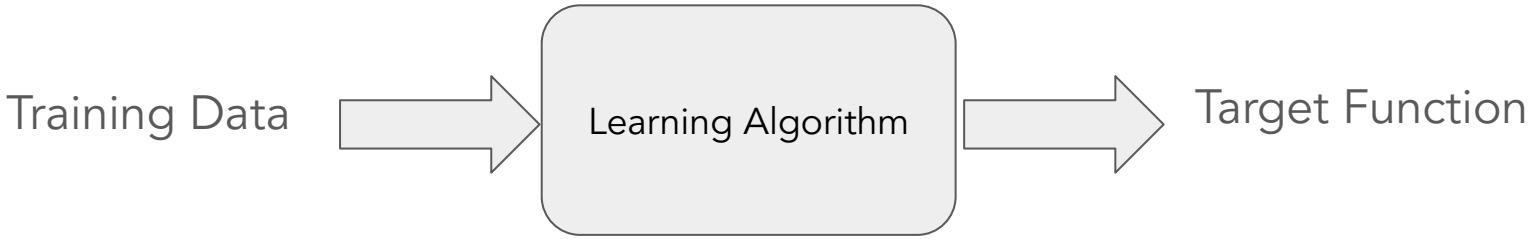


(Large) Language Models

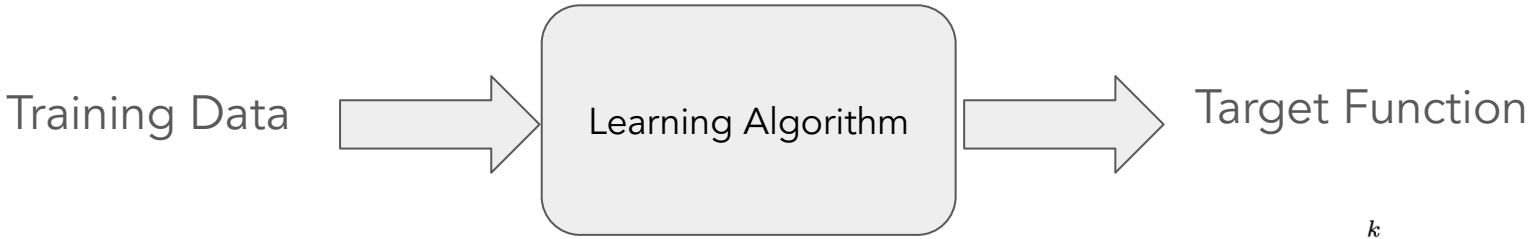


For a movie that gets no respect there sure are a lot of memorable quotes listed for this gem. Imagine a movie where Joe Piscopo is actually funny! Maureen Stapleton is a scene stealer. The Moroni character is an absolute scream. Watch for Alan "The Skipper" Hale jr. as a police Sgt.

$$p(X)$$

$$D = \{X^i\}_{i=1}^n, \quad X^i = \{x_j^i\}_{j=1}^t$$

(Large) Language Models



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$$p(x) = \prod_{i=1}^{k} p(x_i | x_{<i})$$

1. Build probabilistic models
   Categorical Distribution + Autoregressive + RNN->Transformer
2. Derive loss function (by MLE or MAP….)
3. Select optimizer

Combating Combinatorial Complexity: Autoregressive Model

$$p(X) = p\left(\{x_j\}_{j=1}^{t}\right) \qquad O(|V|^T)$$

Combating Combinatorial Complexity: Autoregressive Model

$$p(X) = p\left(\{x_j\}_{j=1}^{t}\right) \qquad O(|V|^T)$$
$$= \prod_{j=1}^{t} p(x_j | x_{<j})$$

Categorical Distribution

$$
\begin{aligned}
p(X) &= p\left(\{x_j\}_{j=1}^{t}\right) && O(|V|^T) \\
&= \prod_{j=1}^{t} p(x_j | x_{<j}) \\
&= \prod_{j=1}^{t} \frac{\exp(W_{x_j}\phi(x_{<j}))}{\sum_{l=1}^{V}\exp(W_l\phi(x_{<j}))}
\end{aligned}
$$

Recursive Neural Network

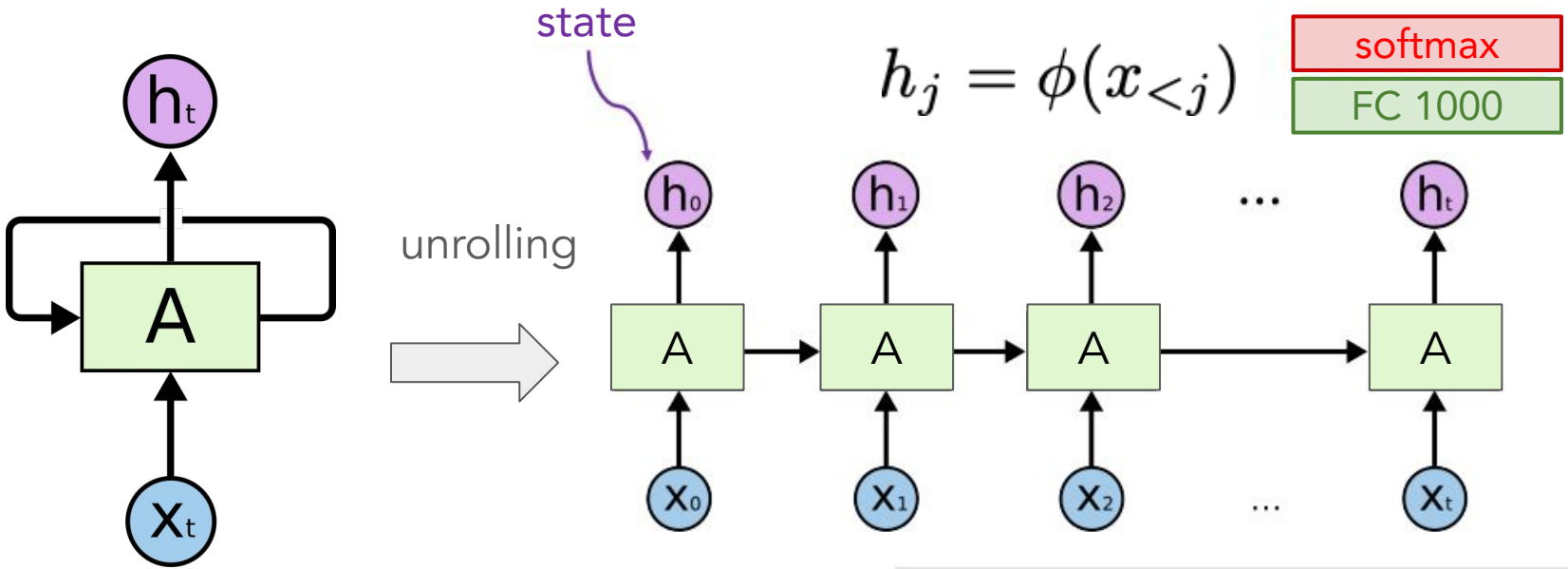
$$p(X) = p\left(\{x_j\}_{j=1}^{t}\right) \qquad O(|V|^T)$$

$$= \prod_{j=1}^{t} p(x_j | x_{<j})$$

$$= \prod_{j=1}^{t} \frac{\exp(W_{x_j} \phi(x_{<j}))}{\sum_{l=1}^{V} \exp(W_l \phi(x_{<j}))} \qquad \text{RNN}$$

Recurrent Neural Network

$$\frac{\exp(W_{x_j}\phi(x_{<j}))}{\sum_{l=1}^{V}\exp(W_l\phi(x_{<j}))}$$



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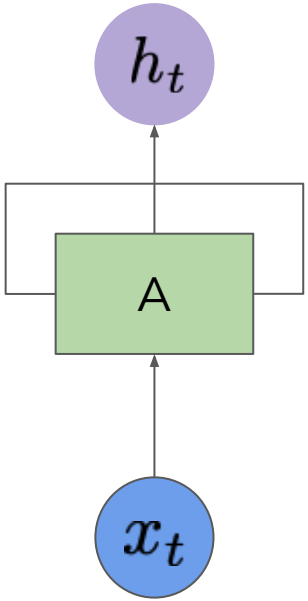
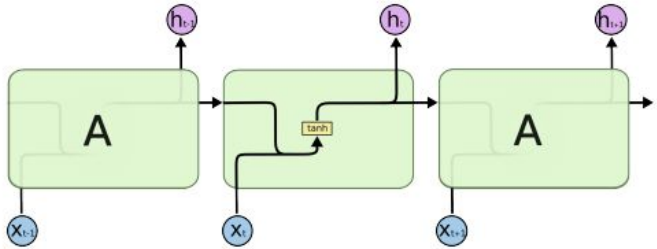
RNN Cell



$$\frac{\exp(W_{x_j}\phi(x_{<j}))}{\sum_{l=1}^{V}\exp(W_l\phi(x_{<j}))}$$

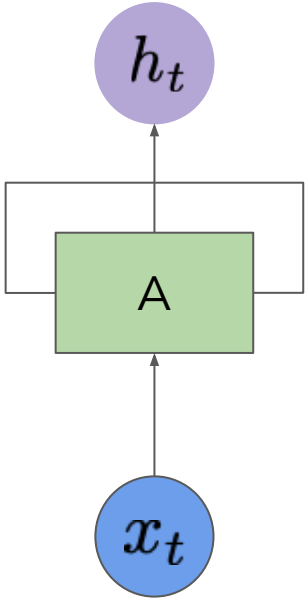


Simple RNN

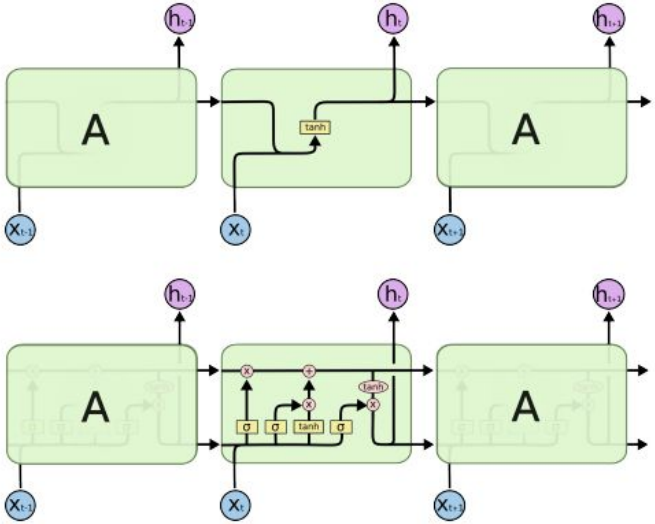
$\mathbf{h_{100}}$ is almost irrelevant to $\mathbf{x_1}$: $\frac{\partial \mathbf{h_{100}}}{\partial \mathbf{x_1}}$ is near zero.

Gradient Vanishing

RNN Cell



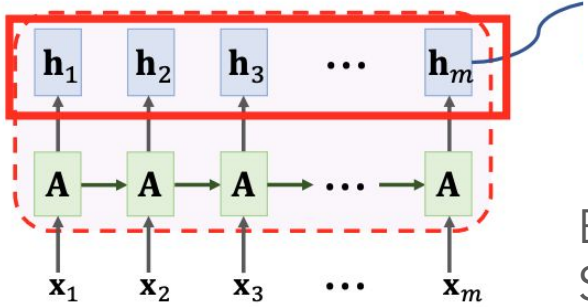
$$\frac{\exp(W_{x_j}\phi(x_{<j}))}{\sum_{l=1}^{V}\exp(W_l\phi(x_{<j}))}$$



Simple RNN

LSTM

Bottleneck in RNN

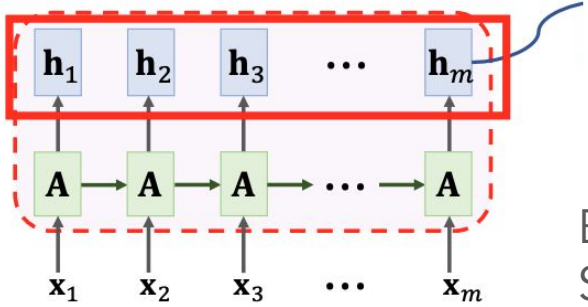


$$h_j = \phi(x_{<j})$$

$$\frac{\exp(W_{x_j}\phi(x_{<j}))}{\sum_{l=1}^{V}\exp(W_l\phi(x_{<j}))}$$

Bottleneck:
Sequences bottlenecked through a fixed-sized vector.

Bottleneck in RNN



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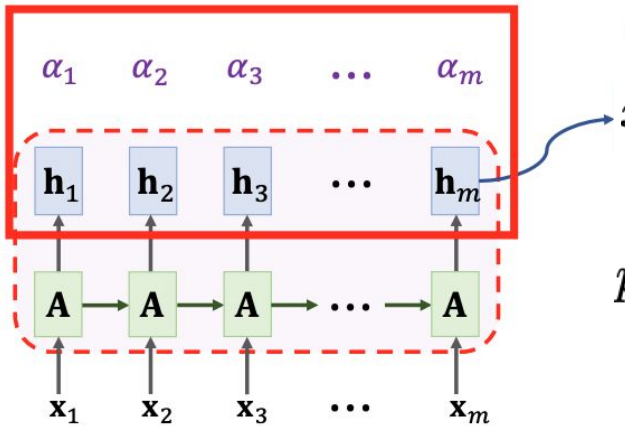
Bottleneck:
Sequences bottlenecked through a fixed-sized vector.

Fixed size -> Length dependent

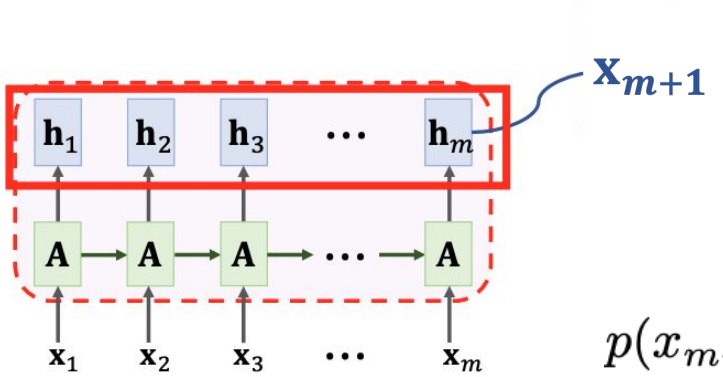
Attention: Flatten RNN Computation

Context vector: $\mathbf{c}_0 = \alpha_1 \mathbf{h}_1 + \cdots + \alpha_m \mathbf{h}_m$.



$$p(x_{m+1}|x_{<m+1}) = \frac{\exp(W_{x_j} c_0)}{\sum_{l=1}^{V} \exp(W_l c_0)}$$

Attention: Adding Output Dependency

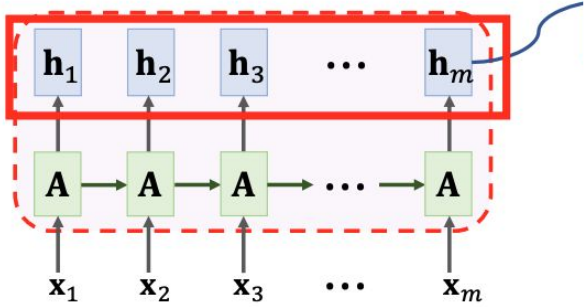


1. Linear maps:
   - $\mathbf{k}_i = \mathbf{W}_K \cdot \mathbf{h}_i$, for $i = 1$ to m.
   - $\mathbf{q}_0 = \mathbf{W}_Q \cdot \mathbf{x}_{m+1}$
2. Inner product:
   - $\tilde{\alpha}_i = \mathbf{k}_i^T \mathbf{q}_0$, for $i = 1$ to m.
3. Normalization:
   - $[\alpha_1, \cdots, \alpha_m] = \text{Softmax}([\tilde{\alpha}_1, \cdots, \tilde{\alpha}_m])$.

$$p(x_{m+1}|x_{<m+1}) = \frac{\exp(W_{x_j} c_0)}{\sum_{l=1}^{V} \exp(W_l c_0)}$$

Weight: $\alpha_i = \text{align}(\mathbf{h}_i, \mathbf{x}_{m+1})$

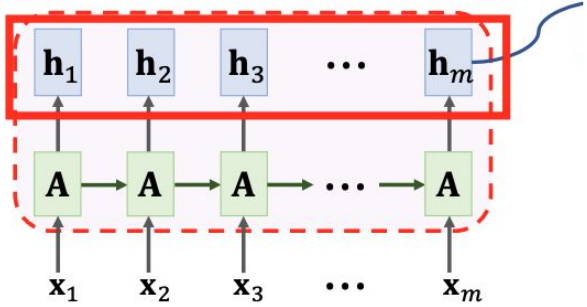
AutoRecursive Attention



$$x_{m+1}\ x_{m+2}, \ldots, x_{m+n}$$

$$p(x_{m+2}|x_{<m+2})$$

AutoRecursive Attention



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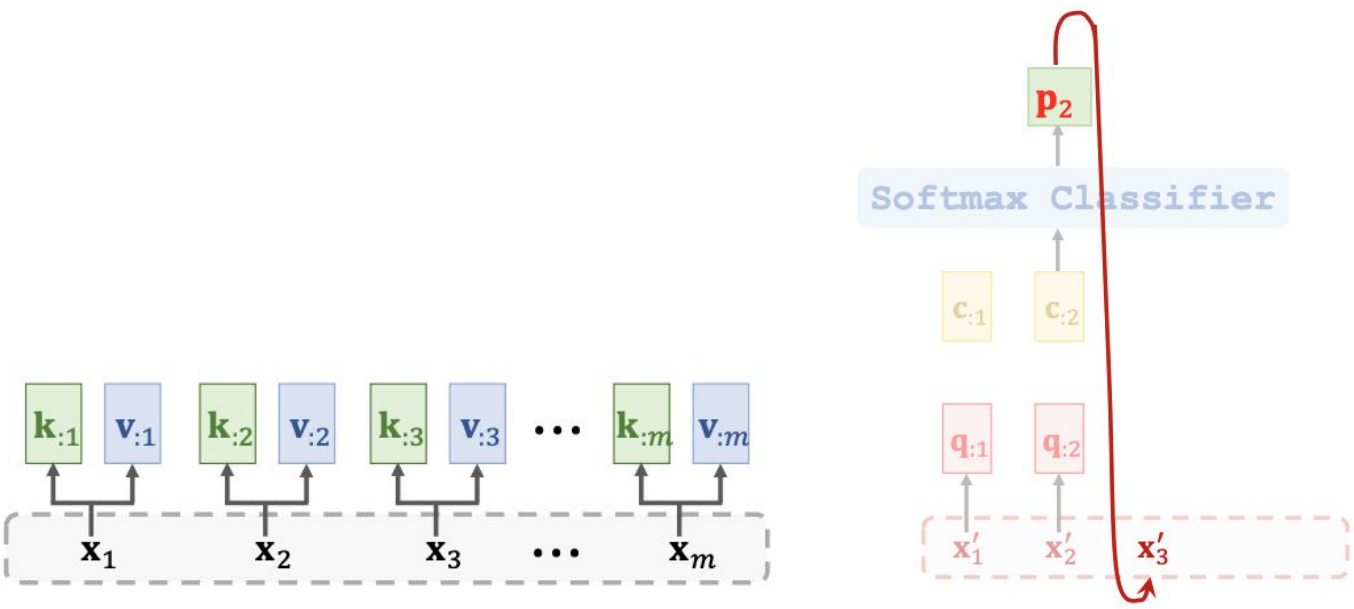
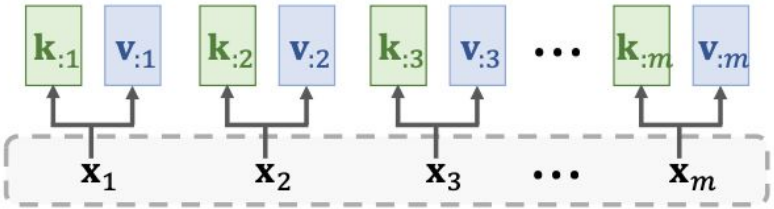
$$p(x_{m+n} | x_{<m+n})$$

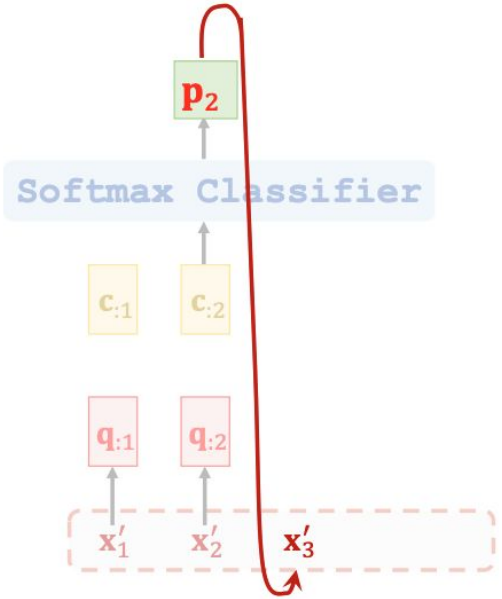
AutoRecursive Attention for Inference



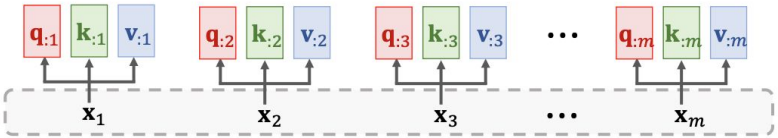
AutoRecursive Attention for Inference

Not flexible enough



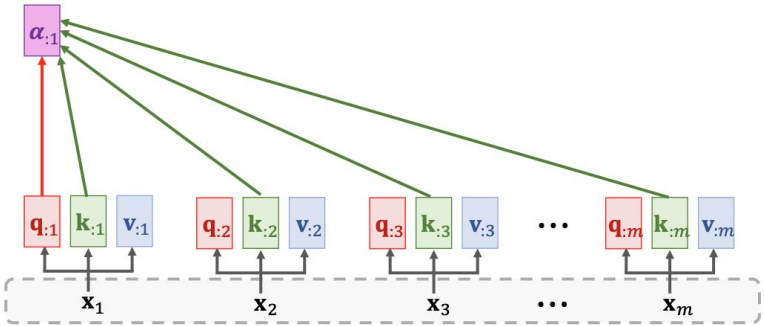


Transformer: Multi-Layer Multi-Headed Self-Attention!



Self-Attention

Weights: $\boldsymbol{\alpha}_{:j} = \text{Softmax}\left(\mathbf{K}^T \mathbf{q}_{:j}\right) \in \mathbb{R}^m.$



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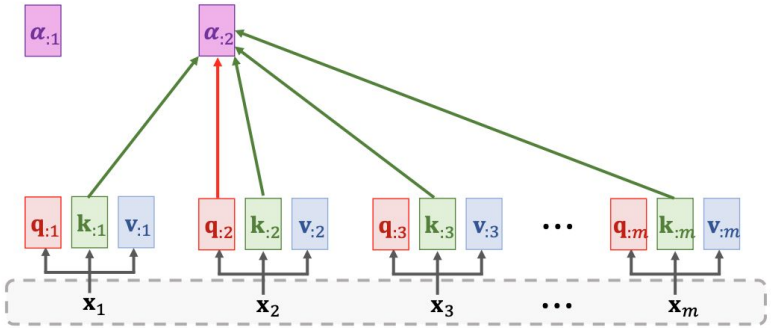

Self-Attention

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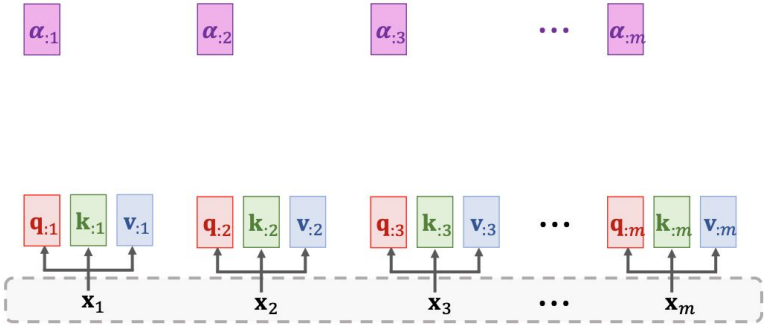
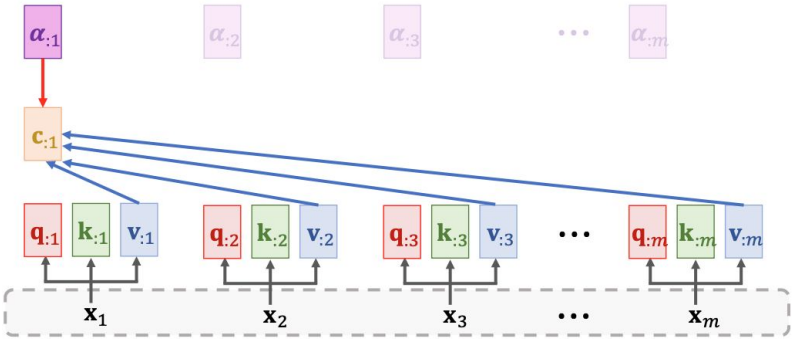


Self-Attention

Context vector: $\mathbf{c}_{:1} = \alpha_{11}\mathbf{v}_{:1} + \cdots + \alpha_{m1}\mathbf{v}_{:m} = \mathbf{V}\boldsymbol{\alpha}_{:1}.$



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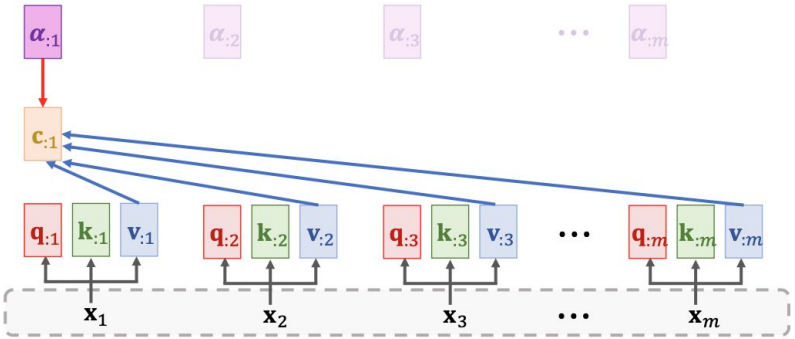

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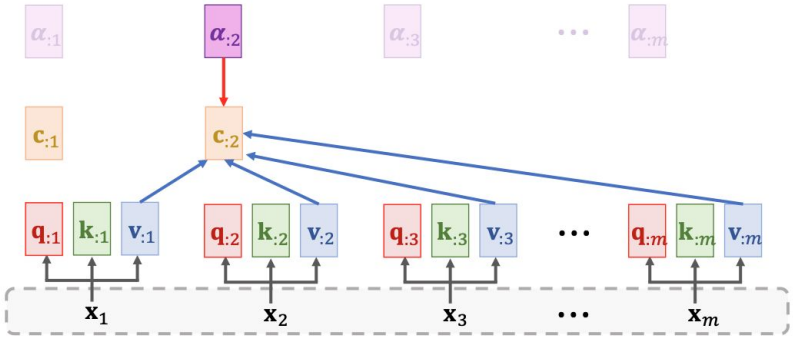

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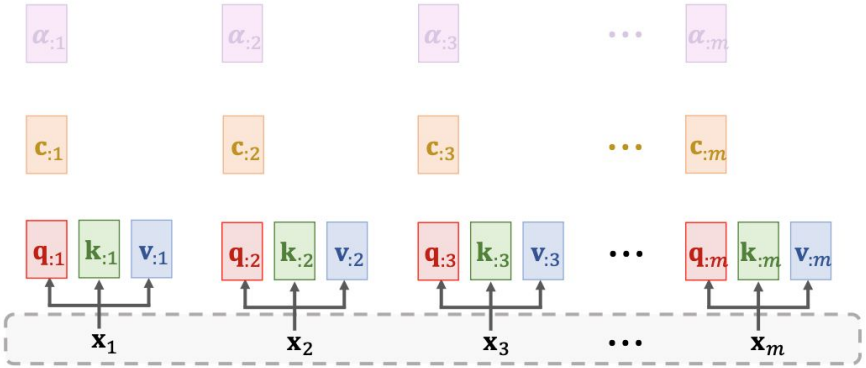

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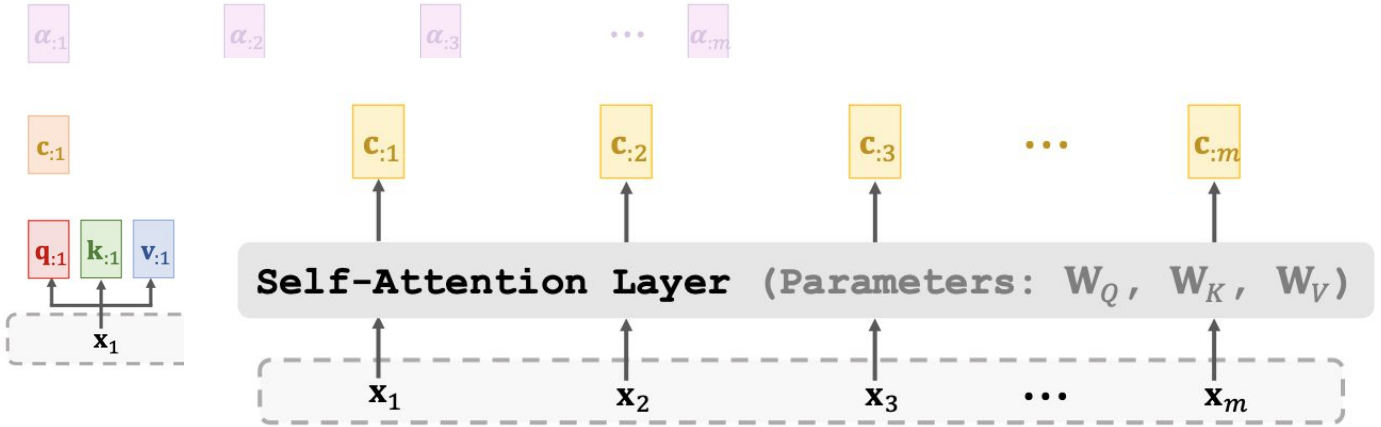
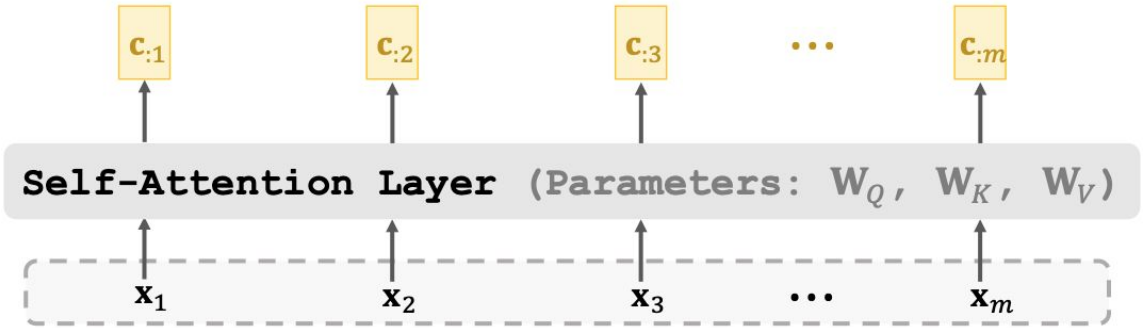
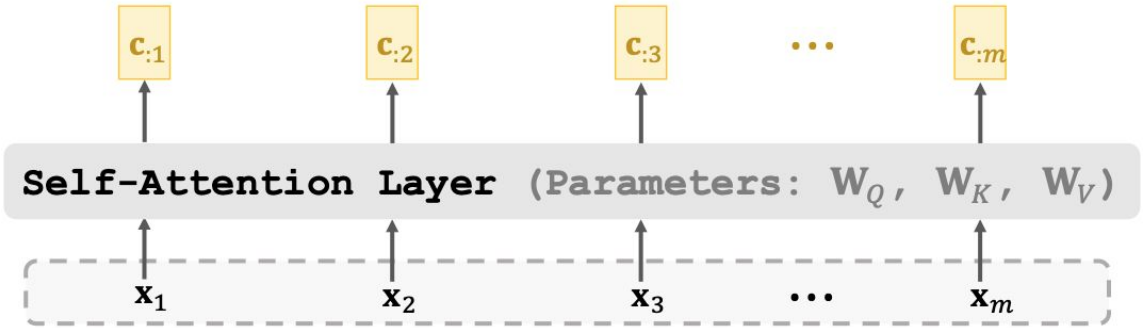


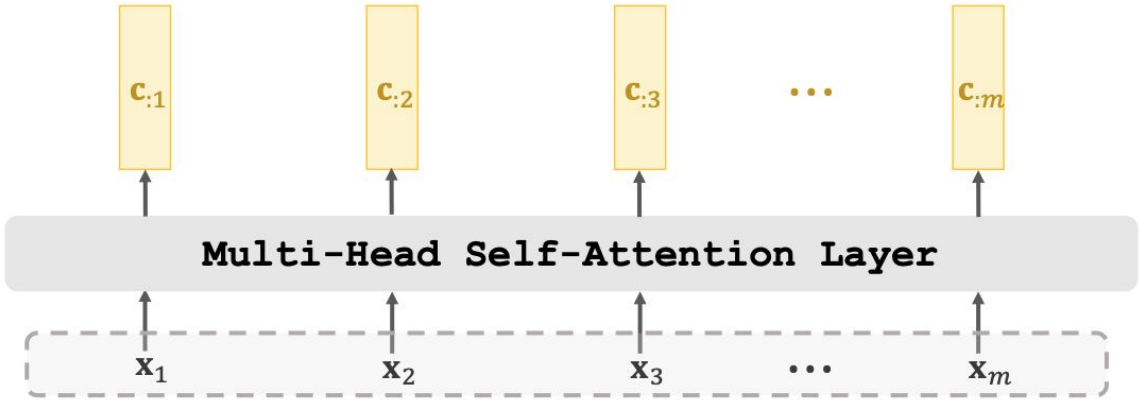
Transformer: Multi-Layer Multi-Headed Self-Attention!



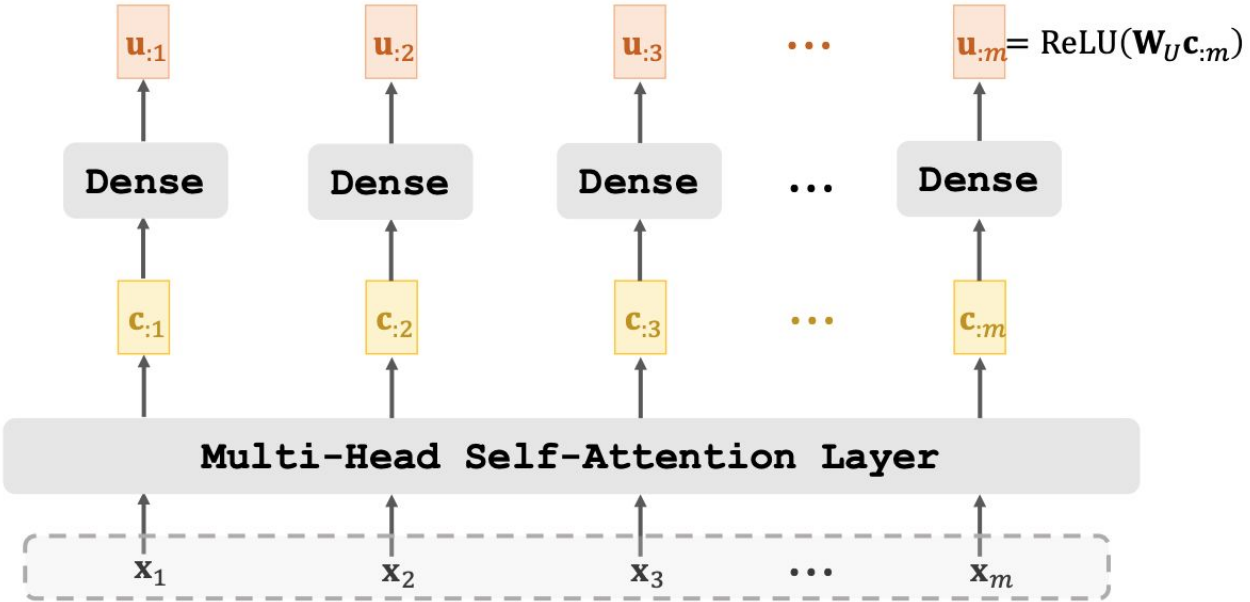
Multi-Headed Self-Attention



Multi-Headed Self-Attention



Multi-Headed Self-Attention



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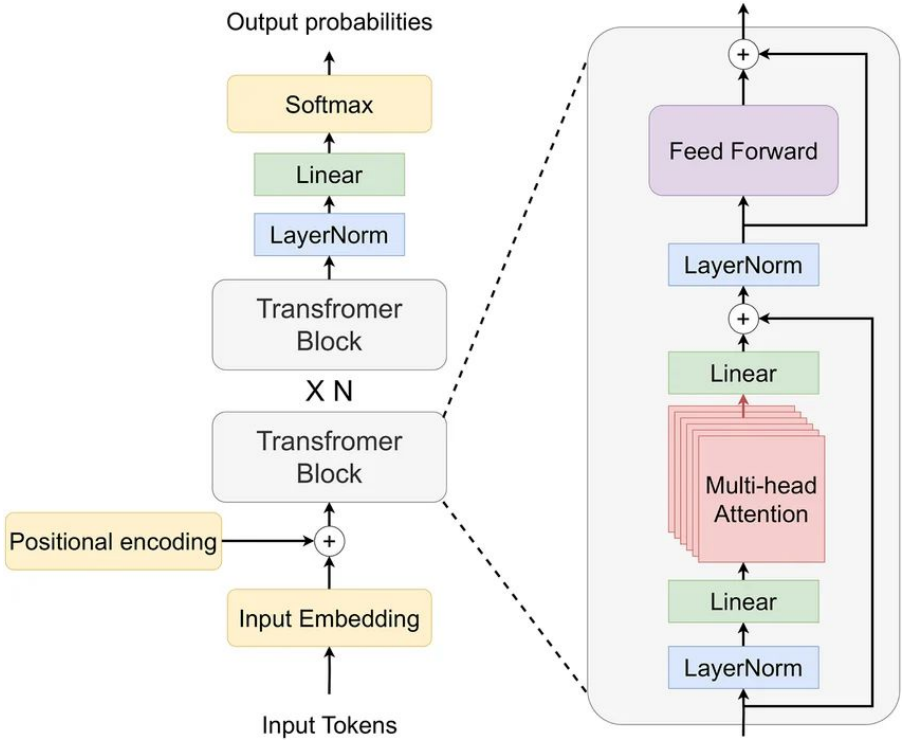
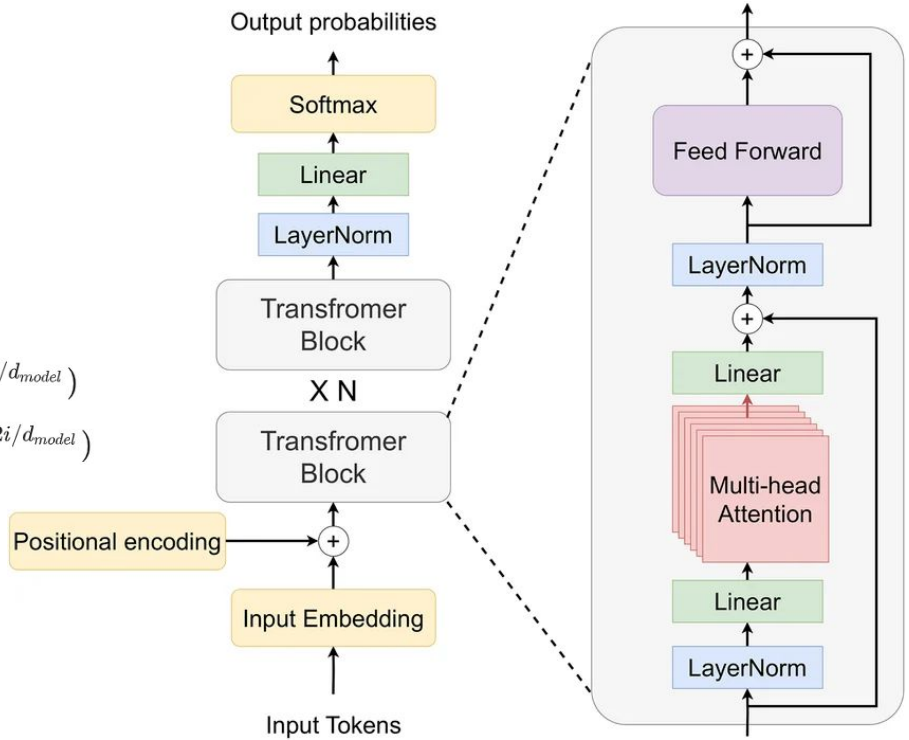


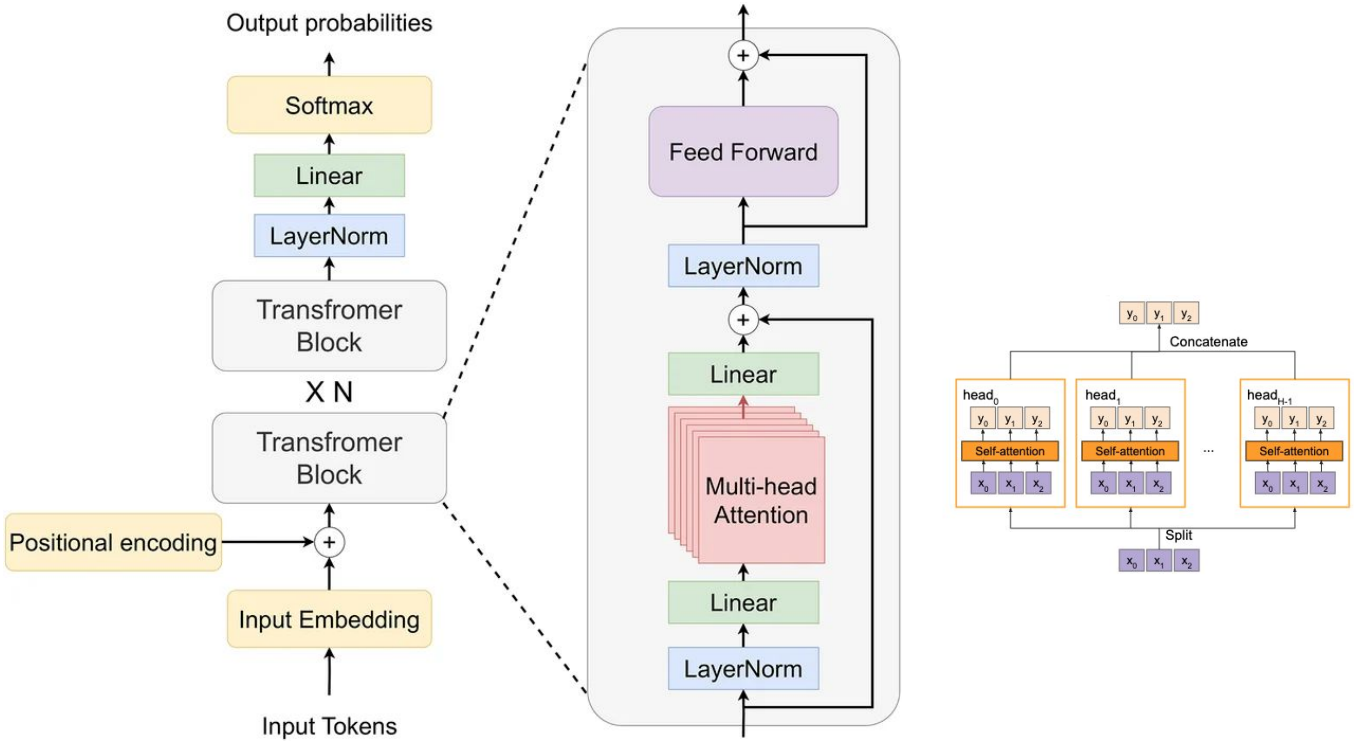
Transformer: Multi-Layer Multi-Headed Self-Attention!

$$PE_{(pos,2i)} = sin(pos/10000^{2i/d_{model}})$$

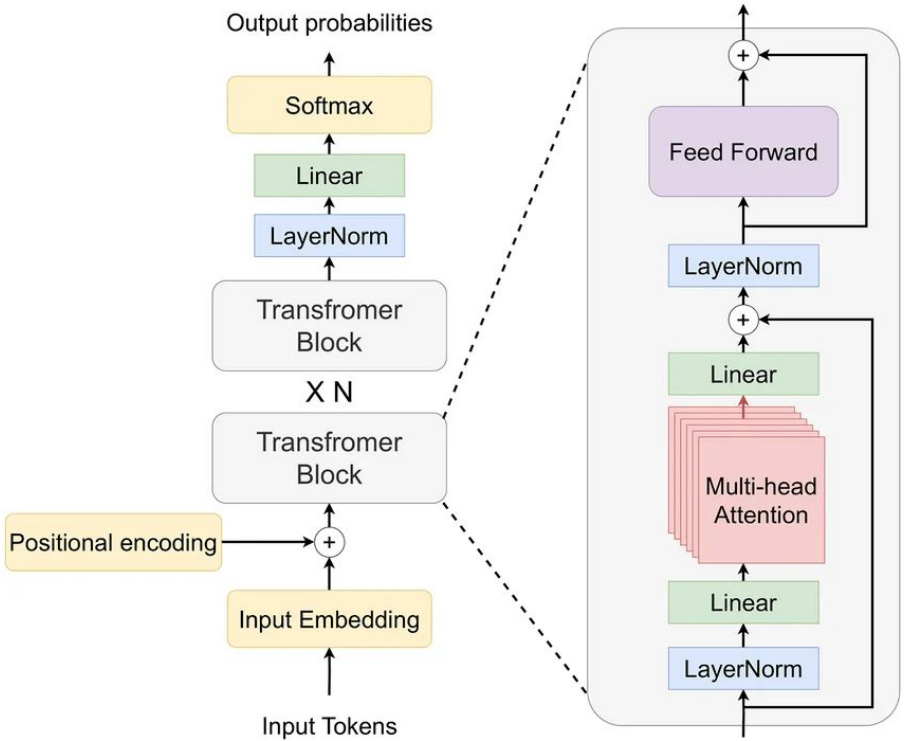
$$PE_{(pos,2i+1)} = cos(pos/10000^{2i/d_{model}})$$



Transformer: Multi-Layer Multi-Headed Self-Attention!



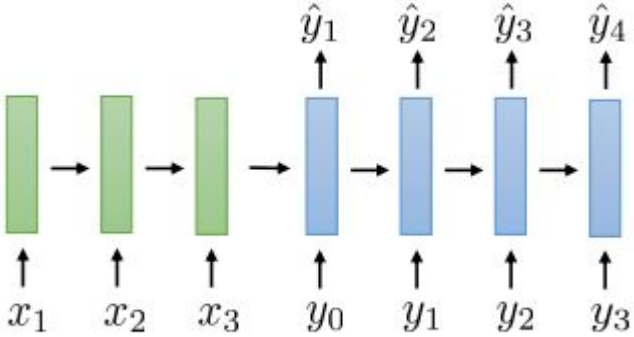
Transformer: Multi-Layer Multi-Headed Self-Attention!

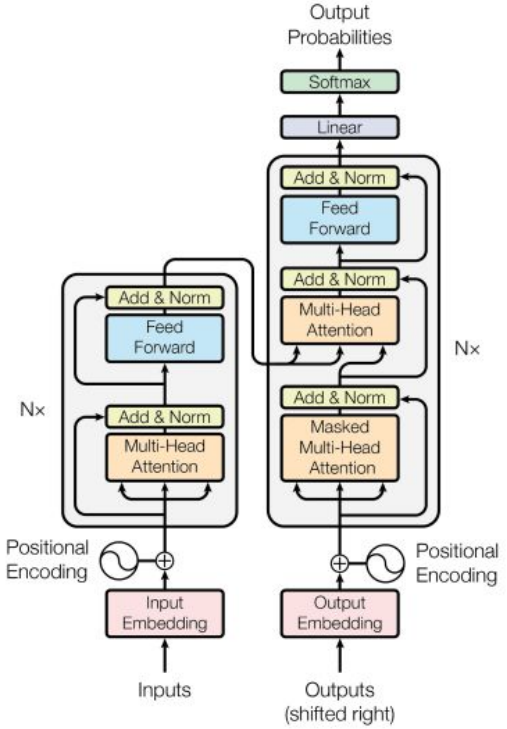


residual connect

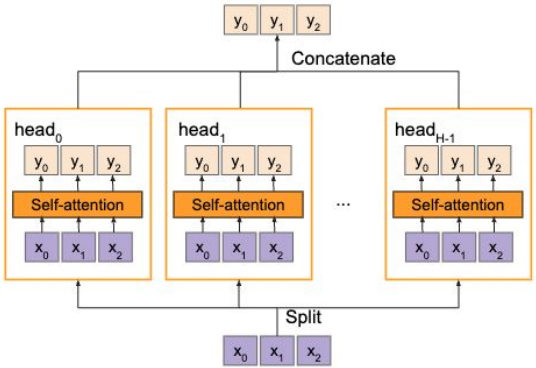
residual connect

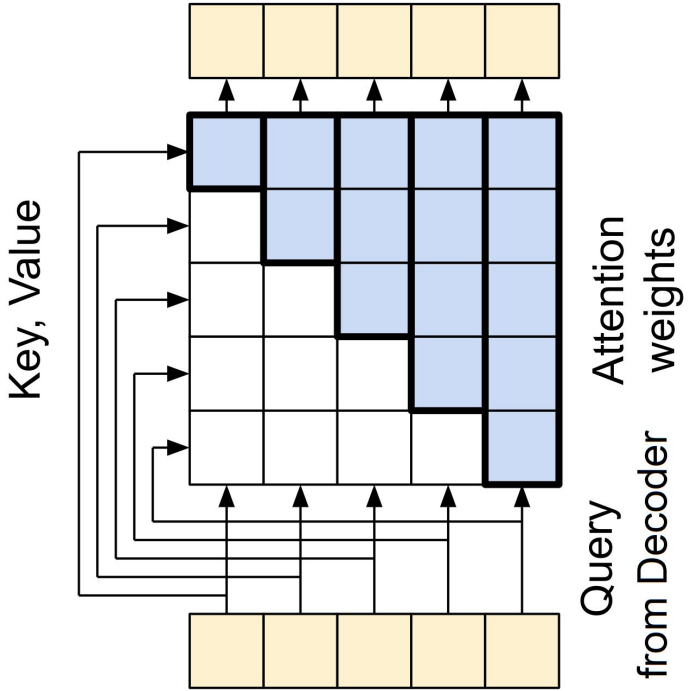
Original Transformer



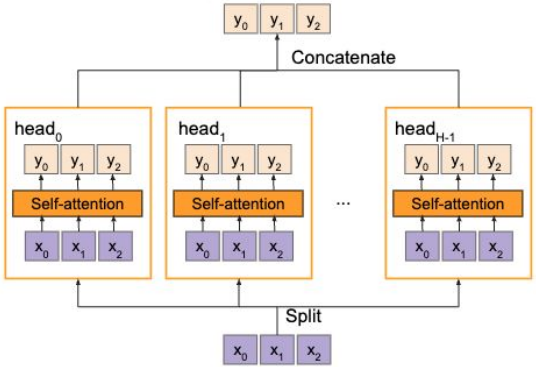


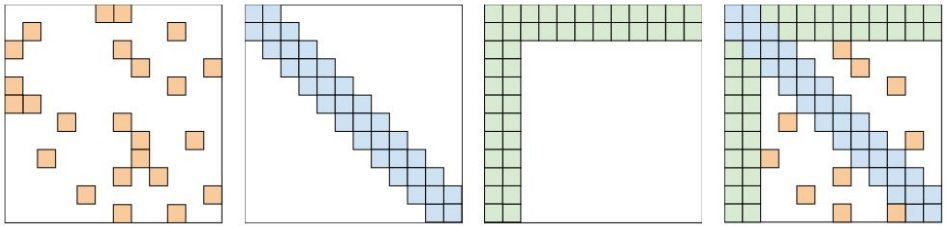
Limitations? Expensive Computation

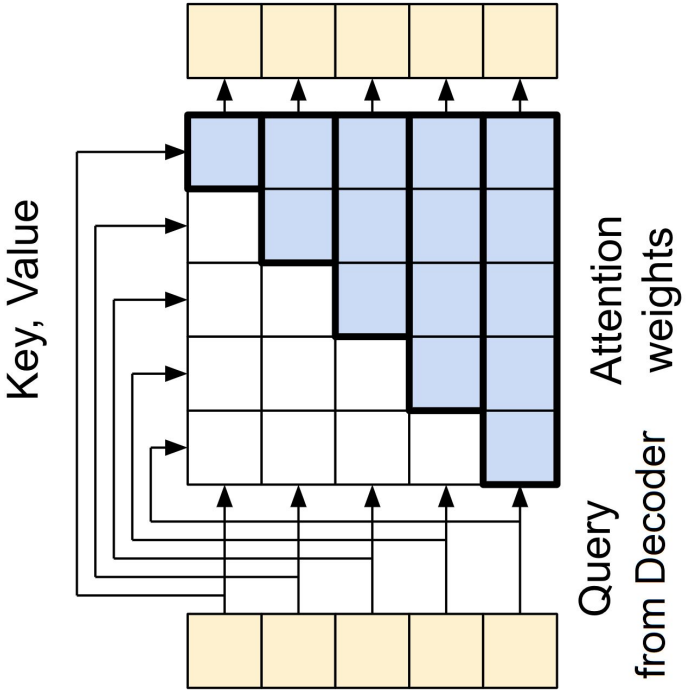




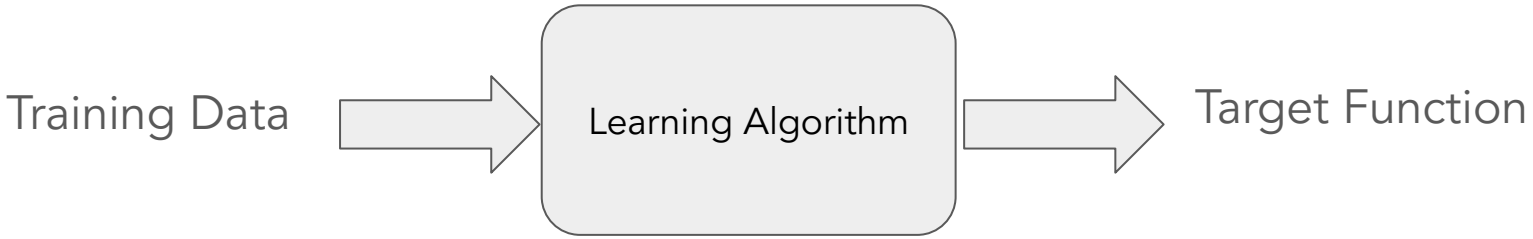
Limitations?




(a) Random attention (b) Window attention (c) Global Attention (d) BIGBIRD



(Large) Language Models



$$D = \{X^i\}_{i=1}^n, \quad X^i = \{x_j^i\}_{j=1}^t$$

$$p(X)$$

1. Build probabilistic models
2. Derive loss function (by MLE or MAP….)
   MLE
3. Select optimizer

MLE

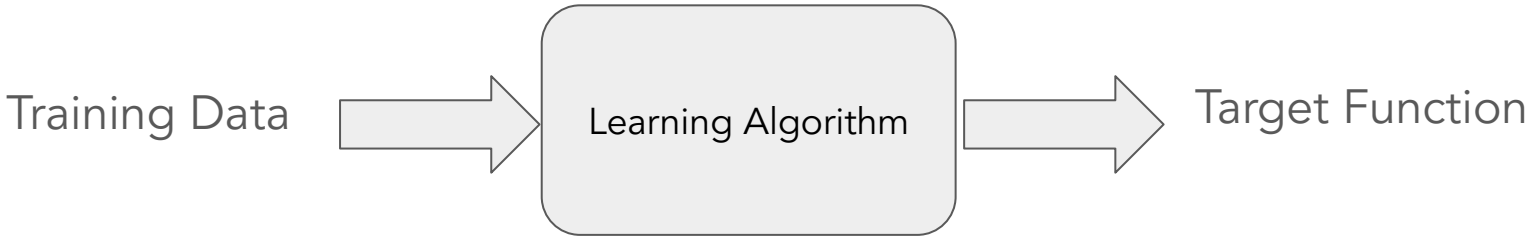
- Given all input data $D = \{X^i\}_{i=1}^n, \quad X^i = \{x_j^i\}_{j=1}^t$

$$p(X^i) = \prod_{j=1}^{t} \frac{\exp\left(W_{x_j^i}\phi\left(x_{<j}^i\right)\right)}{\sum_{l=1}^{V}\exp\left(W_l\phi\left(x_{<j}^i\right)\right)}$$

- Log-likelihood

$$\begin{aligned}
\ell(\phi) &= \sum_{i=1}^{n} p(X^i;\phi) = \sum_{i=1}^{n}\sum_{j=1}^{t}\log\left(x_j^i \mid x_{<j}^i;\phi\right) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{t}\log\frac{\exp\left(W_{x_j^i}\phi\left(x_{<j}^i\right)\right)}{\sum_{l=1}^{V}\exp\left(W_l\phi\left(x_{<j}^i\right)\right)} \\
&= \sum_{i=1}^{n}\sum_{j=1}^{t} W_{x_j^i}\phi\left(x_{<j}^i\right) - \sum_{i=1}^{n}\sum_{j=1}^{t}\log\sum_{l=1}^{V}\exp\left(W_l\phi\left(x_{<j}^i\right)\right)
\end{aligned}$$

(Large) Language Models



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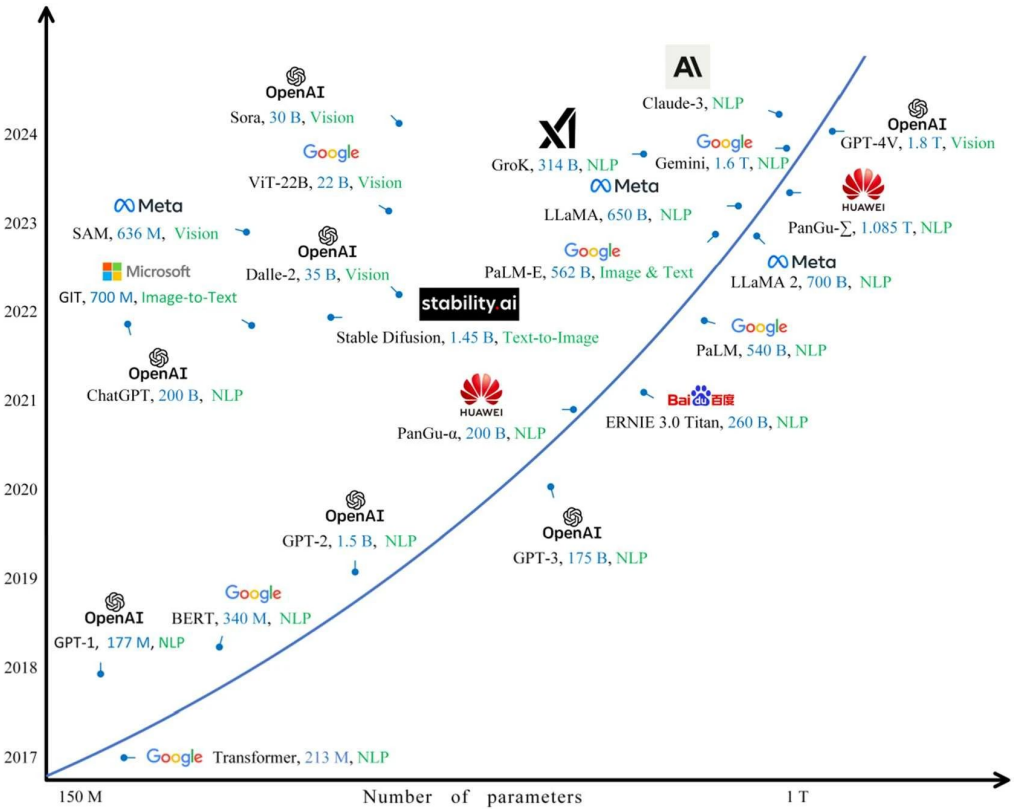
   Stochastic Gradient Descent

Large and Larger LMs



Tu, Xiaoguang, et al. "An overview of large AI models and their applications." *Visual Intelligence* 2.1 (2024): 1-22.

Summary

- Unification is the path to AGI
  - One model for every task

Summary

- Unification is the path to AGI
  - One model for every task
- Pretraining of LLM: MLE for unsupervised learning
- RNNs have bottleneck, restricting the flexibility
- Attention is actually flattening RNN
- Transformer is deep attention

Q&A

Presentation Sign-up deadline today!

Group 7, 11, 18, and 21